Traffic Performance IV: Static and Dynamic Network Supply Models

Ennio Cascetta

Modeling and Simulation of Transportation Networks

July 27, 2015

OUTLINES

• TYPES OF SUPPLY MODELS
  ✓ Classification factors

• STATIC NETWORK MODELS
  ✓ example/definition
  ✓ specification
  ✓ concluding remarks

• WITHIN-DAY DYNAMIC NETWORK MODELS
  ✓ classification
  ✓ example/definition
  ✓ specification

• STATIC vs. DYNAMIC MODELS

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• STATIC vs. DYNAMIC MODELS

TYPES OF SUPPLY SYSTEMS
Classification

• AVAILABILITY OF TRANSPORTATION SERVICES
  ✓ continuous
  ✓ non continuous (scheduled)

• TIME REPRESENTATION AND WITHIN-DAY TEMPORAL VARIABILITY
  ✓ static
  ✓ dynamic
SUPPLY MODELS

Classification factors

• AVAILABILITY OF TRANSPORTATION SERVICES
  ✓ CONTINUOUS
    ➢ services which can be accessed at any location and time
    ➢ examples: car, motorcycle, walking

  ✓ NON CONTINUOUS (SCHEDULED)
    ➢ services which can be accessed at certain locations and times
    ➢ examples: bus, airplane, train

SUPPLY SYSTEM IDENTIFICATION

Classification

• TIME REPRESENTATION
  ANALYSIS PERIOD: the period of time which is considered to be relevant to study a given system

  SIMULATION PERIOD: the period of the simulation relevant for the outputs analysis

  SIMULATION LIFE: the length of the simulation
SUPPLY SYSTEM IDENTIFICATION

Classification

• WITHIN-DAY STATIC

Demand and supply performances constant over subsequent periods

SUPPLY SYSTEM IDENTIFICATION

Classification

• WITHIN-DAY DYNAMIC

Demand and supply performances varying over time
OUTLINES

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• STATIC vs. DYNAMIC MODELS
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STATIC MODELS

Example / definition

• CASE STUDY

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STATIC MODELS
Example / definition

• **GRAPH**

Ordered pair of sets $G(N,L)$ representing network connections

$N \equiv \{1, 2, 3, 4\}$

$L \equiv \{a_1, a_2, a_3, a_4, a_5\}$

Origin centroid: 1

Destination centroid: 4

• **Path $k$:**

Sequence of links representing "typical" trips allowed by the supply system modeled

Path set $K \equiv \{1, 2, 3\}$

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**Link-Path Incidence Matrix**

- $\delta_{ak} = 1$ if link $a$ belongs to path $k$,
- $\delta_{ak} = 0$ otherwise

$\Delta = [\delta_{ak}] = \text{link-path incidence matrix}$

Example:

1st row: link $a_1$ belongs to paths 1 and 3 ($\delta_{a_11} = \delta_{a_13} = 1$)

1st column: path 1 crosses links $a_1$, $a_3$ and $a_5$ ($\delta_{a_11} = \delta_{a_31} = \delta_{a_51} = 1$)
STATIC MODELS

Example / definition

Simulation period = simulation life = 1 hour

• PATH FLOWS

\( \bar{h}_k \) (average) flow along path \( k \) during the simulation period \( T \) [veh/h]

\[ \Rightarrow \bar{h}_1 = 800 \text{ veh/h} \]

\[ \Rightarrow \bar{h}_2 = 1200 \text{ veh/h} \]

\[ \Rightarrow \bar{h}_3 = 400 \text{ veh/h} \]

2.400 veh/h

Path flows are propagated simultaneously on each link belonging to the path

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STATIC MODELS

Example / definition

Network flow propagation

• LINK FLOWS

\( f_{a1} = 1200 \)

\( f_{a2} = 1200 \)

\( f_{a3} = 800 \)

\( f_{a4} = 400 \)

\( f_{a5} = 2000 \)

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• **LINK COST**

\[ c_a \Rightarrow \text{generalized transportation cost of link } a, \text{ i.e. a function of the } n \text{'s disutility (performance) attributes } r_{n,a} \text{ perceived by travelers in making their choices} \]

\[ c_a = c(f) = \sum_n \beta_n r_{n,a}(f) \]

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**STATIC MODELS**

*Example / definition*

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**STATIC MODELS**

*Example / definition*

---

• **LINK COST**

Attributes making up **generalized transportation cost** of link \( a \):

- **Average Travel Time**
  - Running Time
  - Waiting Time

- **Out-of-pocket money**
  - Tolls
  - Gasoline

- **Driving quality**
  (e.g. information systems, etc.)

- **Scenery**
  (e.g. aesthetic attributes)
STATIC MODELS

Example / definition

- LINK PERFORMANCE (1)

Fundamental diagram

Relationships between $v_a$, speed on link $a$, $f_a$, flow on link $a$ and $k_a$, density on link $a$ being

- $v_{a,f}$: free-flow speed (maximum speed)
- $k_{a,jam}$: jam density (storage capacity)
- $Q_a = f_{a,max}$: maximum flow (cross capacity)
- $v_{a,0}$: speed at maximum flow
- $k_{a,0}$: density at maximum flow

![Fundamental Diagram](image)

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STATIC MODELS

Example / definition

- LINK PERFORMANCE (2)

Travel time function

Example: BPR (Bureau of Public Roads)

$$tr'_a(f_a) = \frac{L_a}{v_{0a}} + \gamma_2 \left( \frac{L_a}{v_{ca}} - \frac{L_a}{v_{la}} \right) \left( \frac{f_a}{Q_a} \right)^{\gamma_2}$$

being $\gamma_1$ and $\gamma_2$ two parameters to be calibrated

![Travel Time Function](image)

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STATIC MODELS

Example / definition

• LINK PERFORMANCE (2)

Waiting Time functions at signalized intersections

Example: *Webster’s two terms formula*

\[
t_{wa}(f_a) = 0.9 \left( \frac{T_c}{2(1 - f_a/S_a)} + \frac{(f_a/Q_a)^2}{2f_a(1 - f_a/Q_a)} \right)
\]

- \(T_c\) is the cycle length
- \(\mu\) is the effective green to the cycle length ratio
- \(f_a\), \(Q_a\) are the flow and the capacity of the lane group on link \(a\), respectively

\[
G = 60 \text{ sec} \\
T_c = 120 \text{ sec} \\
S = 3600 \text{ veh/h} = 1 \text{ veh/sec} \\
G/T_c = 0.5 \\
\alpha = 0.95 \\
Q = 3800 \text{ veh/h} = 0.5 \text{ veh/sec}
\]

![Diagram](image)

\[\text{TT}_{a1} = 8.2 \quad \text{TT}_{a2} = 12.1 \quad \text{TT}_{a3} = 8.7 \quad \text{TT}_{a4} = 5.6 \quad \text{TT}_{a5} = 12.5\]

STATIC MODELS

Example / definition

• AVERAGE LINK TRAVEL TIMES

BPR parameters: \(\gamma_1 = 1 \quad \gamma_2 = 2\)

\[
tr_a(f_a) = \frac{L_a}{v_{a,\theta}} + \gamma_2 \left( \frac{L_a}{v_{a,c}} - \frac{L_a}{v_{a,c}} \right) \left( \frac{f_a}{Q_a} \right)^{\gamma_2}
\]

<table>
<thead>
<tr>
<th>links</th>
<th>(L_a)</th>
<th>(v_{a,\theta})</th>
<th>(v_{a,c})</th>
<th>(Q_a)</th>
<th>(f_a)</th>
<th>(tr_a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_1)</td>
<td>6.0</td>
<td>60</td>
<td>30</td>
<td>2000</td>
<td>1200</td>
<td>8.2</td>
</tr>
<tr>
<td>(a_2)</td>
<td>8.9</td>
<td>60</td>
<td>30</td>
<td>2000</td>
<td>1200</td>
<td>12.1</td>
</tr>
<tr>
<td>(a_3)</td>
<td>7.5</td>
<td>60</td>
<td>30</td>
<td>2000</td>
<td>800</td>
<td>8.7</td>
</tr>
<tr>
<td>(a_4)</td>
<td>7.4</td>
<td>80</td>
<td>40</td>
<td>4000</td>
<td>400</td>
<td>5.6</td>
</tr>
<tr>
<td>(a_5)</td>
<td>6.3</td>
<td>60</td>
<td>30</td>
<td>2000</td>
<td>2000</td>
<td>12.5</td>
</tr>
</tbody>
</table>

*Ennio Cascetta - Static and Dynamic Network Supply Model*
STATIC MODELS

Example / definition

• LINK COSTS

\[ c_{a1} = 8.2 \text{ min} \]
\[ c_{a2} = 12.1 \text{ min} \]
\[ c_{a3} = 8.7 \text{ min} \]
\[ c_{a4} = 5.6 + \left( \frac{2}{5} \right) \times 60 = 29.6 \text{ min} \]

VOT = 5 €/h
Toll = 2 €

\[ c_{a5} = 12.5 \text{ min} \]

• PATH COSTS

\[ g_k \rightarrow \text{cost of the path } k \]

\[ g_1 = c_{a1} + c_{a3} + c_{a5} = 29.5 \text{ min} \]

\[ g_2 = c_{a2} + c_{a5} = 24.6 \text{ min} \]

\[ g_3 = c_{a1} + c_{a4} = 37.8 \text{ min} \]

Path costs are not always additive w. r. t. link costs!
STATIC MODELS

Specification

Set of equations relating path costs and other intermediate variables (link flows, travel times, etc.) to path flows

taken from:

OVERALL SUPPLY MODEL

GRAPH MODEL
\[(N, L) ; \Delta\]

NETWORK FLOW PROPAGATION MODEL
\[f = \Delta \cdot h\]

LINK PERFORMANCE MODEL
\[c_a = c(f) = \sum \beta_n \cdot r_{n,a}(f)\]

PATH PERFORMANCE MODEL
\[g = \Delta^T \cdot c\]

\[g = \Delta^T \cdot c(\Delta \cdot h)\]
STATIC MODELS

Specification

O-D = (1,4)

Paths:

Path performance model
\[ g = \Delta^T \]

Network flow propagation model
\[ f = \Delta \]

\[
\begin{bmatrix}
29.5 \\
24.6 \\
37.8
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
8.2 \\
12.1 \\
29.6 \\
12.5
\end{bmatrix}
\]

\[
\begin{bmatrix}
1200 \\
1200 \\
800 \\
400 \\
2000
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 1 \\
1 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
800 \\
1200 \\
400
\end{bmatrix}
\]

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STATIC MODELS

Concluding remarks

“Static outputs”:
The functioning of the system along the simulation period is captured by a “picture” of the system

Examples of typical outputs of a static model:

Link flows

Path costs

\[ g_1 = c_{a1} + c_{a3} + c_{a4} \]

\[ g_2 = c_{a2} + c_{a4} \]

\[ g_3 = c_{a3} + c_{a5} \]

“Continuous flows”:
users are undistinguishable particles of a fluid

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STATIC MODELS

The “Art” Of Building Network Models

- study area and zoning
- base network extraction
- graph specification
- cost functions

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STATIC MODELS

The “Art” Of Building Network Models

- study area and zoning
- Internal and external “traffic zones”
- **Centroids**: ideal points assumed to be the origin (destination) of every trip starting from (arriving to) a certain traffic zone

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STATIC MODELS
The “Art” Of Building Network Models

Criteria for the definition of the study area

Criteria for zoning
- Traffic zones are often obtained as aggregations of administrative areas (e.g. census sections, municipalities or provinces).
- Physical geographic separators (e.g. rivers, railway lines) are conventionally used as zone boundaries.
- A different level of zoning detail can be adopted for different parts of the study area depending on the precision needed.
- Traffic zones should aggregate parts of the study area which are “homogeneous” with respect both to their land-use (e.g. residential or commercial zones) and to their accessibility to transportation facilities and services.

A larger number of zones:
- more precise representation of the real system
- increases the computational burden

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STATIC MODELS
The “Art” Of Building Network Models

- base network extraction
  Consistency between zoning and network detail

STATIC MODELS
Concluding remarks

THE “ART” OF BUILDING NETWORK MODELS

- graph specification
  - Level of details depends on the scope of the simulation
  - Example: representation of an intersection
OUTLINES

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WITHIN-DAY DYNAMIC MODELS

Classification

Continuous Flow
• Users are simulated as indistinguishable particles of a mono-
dimensional partly compressible fluid
• Vehicle trajectories are not explicitly traced
  ✓ Discrete space MACRO-SIMULATION MODELS (also referred as
    ANALYTICAL models)
  ✓ Continuous space MACRO-SIMULATION MODELS

Discrete Flow
• Users are simulated as discrete units, single vehicles or packet of
  vehicles sharing the same trip
• Vehicle trajectories are explicitly traced
• MESO-SIMULATION MODELS (Aggregate, i.e. link-based, performance
  functions)
• MICRO-SIMULATION MODELS (Disaggregate performance functions)
WITHIN-DAY DYNAMIC MODELS
Classification

<table>
<thead>
<tr>
<th>Flow Representation</th>
<th>Performance functions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AGGREGATE (explicit capacity)</td>
</tr>
<tr>
<td>CONTINUOUS</td>
<td>MACRO-SIMULATION</td>
</tr>
<tr>
<td></td>
<td>discrete space</td>
</tr>
<tr>
<td>DISCRETE</td>
<td>MESO-SIMULATION</td>
</tr>
</tbody>
</table>

Example / definition

Packet Approach
A group of users leaving during the same interval and following the same path (in the limit, individual vehicles)

FEATURES:
- some detailed simulation possibilities (such as spill-back or while-trip re-routing)
- applicable to general networks
WITHIN-DAY DYNAMIC MODELS

Example / definition

Notation and terminology

- **TIME VARIABLES**
  - $T$ the simulation period
  - $j$ the generic time interval within the simulation period
  - $DT$ duration of interval $j$
  - $\tau_j$ the characteristic instant for time interval $j$ (e.g. end-point of the time interval)
  - $\Delta t$ the simulation step (i.e. the time unit of the simulation clock); typically $\Delta t << DT$

- **FLOW VARIABLES**
  - $k_j$ the packet along path $k$ departing during $j$ (at the characteristic instant $\tau_j$)

---

**Example** (same network of the static case)
- Simulation period $T$: one hour
- Simulation life: two hours
(i.e. simulation continues until all the packets departed during simulation period reach their destination)
- Number of time intervals: four $\rightarrow j = 1, 2, 3, 4$
- Interval length: $\rightarrow DT = 15$ minutes
- Characteristic instant for time interval $j$: $\tau_j = 15, 30, 45, 60$
- End-point of $j$: $\rightarrow \Delta t = 30$ sec
- Single origin-destination pair (1-4)
- No capacity restraints (no spill-back)

- Packets $k_j$ departing during the simulation period
  - at the characteristic instant $\tau_j$)

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WITHIN-DAY DYNAMIC MODELS

Example / definition

Path flows (time dependent) (1/2)

\[ h_{k}[j] \]

number of vehicles making up a packet \( k_j \) (packet dimension [veh])

\[ \bar{h}_k = \frac{1}{T} \sum_{j} h_{k}[j] \]

average flow along path \( k \) departing during simulation period \( T \) [veh/h]

WITHIN-DAY DYNAMIC MODELS

Example / definition

Path flows (time dependent) (2/2)

Path \( k=1 \)

\[ h_1 = \begin{bmatrix} 
200 & t_1 \\
100 & t_2 \\
400 & t_3 \\
100 & t_4 
\end{bmatrix} \]

\[ \bar{h}_1 = 800 \text{ veh/h} \]

Path \( k=2 \)

\[ h_2 = \begin{bmatrix} 
300 & 2_1 \\
300 & 2_2 \\
200 & 2_3 \\
400 & 2_4 
\end{bmatrix} \]

\[ \bar{h}_2 = 1200 \text{ veh/h} \]

Path \( k=3 \)

\[ h_3 = \begin{bmatrix} 
0 & 3_1 \\
100 & 3_2 \\
300 & 3_3 \\
0 & 3_4 
\end{bmatrix} \]

\[ \bar{h}_3 = 400 \text{ veh/h} \]
WITHIN-DAY DYNAMIC MODELS

Example / definition

Path flows (time dependent)

Network flow propagation

Link flows (time dependent)

Time-varying link flows across sections depend on the times at which path flows (packets) reach that section. Moreover, the link flow in a specific interval depends on the section s of the link:

\[ \sum_{k} f_{a,s}^{k}(j) \]

\[ f_{a,s}^{k}(j) \] number of users on path k crossing section s of link a during time interval j

\[ f_{a,s}^{k}(j) = \sum_{k} f_{a,s}^{k}(j) \]

Total flow crossing section s of link a during interval j

“Characteristic” link flows

\[ u_{a}^{k}(j) = f_{a,s=0}^{k}(j) \]

number of users on path k entering link a during time interval j (inflow)

\[ w_{a}^{k}(j) = f_{a,s=L_a}^{k}(j) \]

number of users on path k exiting link a during time interval j (outflow)

\[ u_{a}(j) = \sum_{k} u_{a}^{k}(j) \]

Total inflow on link a during interval j

\[ w_{a}(j) = \sum_{k} w_{a}^{k}(j) \]

Total outflow on link a during interval j
WITHIN-DAY DYNAMIC MODELS

Example / definition

• Example: total inflow  \( u_a[j] = \sum_k u_a^k[j] \)

\[ \begin{align*}
\tau_1 &= 15 \\
\tau_2 &= 30 \\
\tau_3 &= 45 \\
\tau_4 &= 60
\end{align*} \]

\[ \begin{align*}
u[j=1] &= \begin{bmatrix} 0 \\ 300 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\
u[j=2] &= \begin{bmatrix} 0 \\ 300 \\ 0 \\ 300 \\ 0 \\ 0 \end{bmatrix} \\
u[j=3] &= \begin{bmatrix} 700 \\ 200 \\ 100 \\ 0 \\ 0 \\ 300 \end{bmatrix} \\
u[j=4] &= \begin{bmatrix} 100 \\ 400 \\ 0 \\ 0 \\ 300 \end{bmatrix}
\]

note: in the pictures, each unit represents 100 vehicles

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WITHIN-DAY DYNAMIC MODELS

Example / definition

• Example: total outflow  \( w_a[j] = \sum_k w_a^k[j] \)

\[ \begin{align*}
\tau_1 &= 15 \\
\tau_2 &= 30 \\
\tau_3 &= 45 \\
\tau_4 &= 60
\end{align*} \]

\[ \begin{align*}
w[1] &= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\
w[2] &= \begin{bmatrix} 200 \\ 300 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\
w[3] &= \begin{bmatrix} 200 \\ 300 \\ 200 \\ 0 \\ 0 \end{bmatrix} \\
w[4] &= \begin{bmatrix} 0 \\ 200 \\ 100 \\ 100 \\ 500 \end{bmatrix}
\]

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**WITHIN-DAY DYNAMIC MODELS**

*Example / definition*

\[ U_a(\tau_j) \] Cumulated number of vehicles arrived to link \( a \) up to time \( \tau_j \) \[ = \sum_{j'} u_a[j'] \] (cumulated inflow)

\[ W_a(\tau_j) \] Cumulated number of vehicles which have left link \( a \) up to time \( \tau_j \) \[ = \sum_{j'} w_a[j'] \] (cumulated outflow)

\[ n_a(\tau_j) = U_a(\tau_j) - W_a(\tau_j) \]

Number of users on link \( a \) at the characteristic instant \( \tau_j \)

(\text{load} = n_a(\tau_j) = k_a(\tau_j) \cdot L_a)

\[ n_a(\tau_j) = n_a(\tau_{j-1}) + u_a[j] - w_a[j] \]

---

**WITHIN-DAY DYNAMIC MODELS**

*Example / definition*

- Example: loads \( n_a(\tau_j) = U_a(\tau_j) - W_a(\tau_j) \)

- \( \tau_1 = 15 \)

- \( \tau_2 = 30 \)

- \( \tau_3 = 45 \)

- \( \tau_4 = 60 \)

---

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WITHIN-DAY DYNAMIC MODELS

Example / definition

Flows consistency on the network

1. NODE FLOW CONSERVATION (along path $k$)

\[ u^k_{a_1} [j] = h^k_k [j] \]  \hspace{1cm} (path flows)  \hspace{1cm} (eq.1)

where $u^k_{a_1}$ is the inflow and $h^k_k$ is the outflow of path $k$.

2. LINK FLOW CONSERVATION

\[ u^k_{a_{i+1}} = w^k_{a_i} \]  \hspace{1cm} (eq.2)

\[ n_a(\tau_j) = U_a(\tau_j) - W_a(\tau_j) \]  \hspace{1cm} (loads)  \hspace{1cm} (eq.3)

\[ n_a(\tau_j) = n_a(\tau_{j-1}) + u_a[j] - W_a[j] \]  \hspace{1cm} (eq.4)
WITHIN-DAY DYNAMIC MODELS

Example / definition

Link performances (1)

• TRAVEL TIME VARIABLES

\[ t^f_a(\tau) \text{ Link “forward” travel time:} \]
travel time to cross link \( a \) entering link \( a \) at time \( \tau \)

**Note:** Travel time to cross a link changes during the simulation period!

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WITHIN-DAY DYNAMIC MODELS

Example / definition

Link performances (2)

• TRAVEL TIME FUNCTIONS

Give the forward travel time \( t^f_{a,j}(j) \) of each packet entering link \( a \) at a generic time instant of interval \( j \), as a function of, for instance:

Link density (load) at the characteristic time instant \( \tau_{j-1} \) of the previous interval \( j-1 \)

**Example:** “Steady state” Greenshield travel time function becomes:

\[ v_a(j) = v_{a,0} \left( 1 - \frac{k_{a}(\tau_{j-1})}{k_{a,\text{max}}} \right) \]
\[ t^f_{a,j}(j) = \frac{L_a}{v_a(j)} = t^f_{a,0} \left( \frac{n_{\text{max}} - n_a(\tau_{j-1})}{n_{\text{max}} - n_a(\tau_{j-1})} \right) \]

being \( n_{\text{max}} = k_{a,\text{max}} \cdot L_a \) the maximum load on link \( a \)

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WITHIN-DAY DYNAMIC MODELS

Example / definition

Link performances (1)
As time-varying path flows propagate on the network inducing time-varying link loads, link travel times vary along the simulation period.

For instance, assuming that simulation starts with the network “empty” and terminates when it’s again “cleaned up”:
• at the beginning, link travel times are equal to free-flow travel times;
• at the end, link travel times are equal again to free-flow travel times;
• at any time during simulation period, link travel times depend upon link loads at that instant.

Link performances (2)

![Link Travel Times](image)

Link Travel Times

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WITHIN-DAY DYNAMIC MODELS

Example / definition

Dynamic incidence matrix (1)
The dynamic incidence matrix maps path flows departed in interval $l$ to inflows entering link $a$ in interval $j$

$$\Delta[l, j] = [\delta_{ak}(l, j)]_{ak} \quad l \leq j$$

$$\delta_{ak}[l, j] = \begin{cases} 1 & \text{if } \tau^u_k \in ([j-1]DT, [j]DT) \\ 0 & \text{otherwise} \end{cases}$$

i.e. the dynamic incidence value $\delta_{ak}(l, j)$ tells us if packet $k$ has entered link $a$ during time interval $j$.

$\delta_{ak}(l, j)$ is a function of the travel times on the links preceding $a$ on path $k$.

Dynamic incidence matrix (2)
The dynamic incidence value $\delta_{ak}(l, j)$ is a function of the travel times on the links preceding $a$ on path $k$.

$$\delta_{ak}[l, j] = \Gamma[t_{a' \in k}^f(i), l \leq i \leq j]$$

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WITHIN-DAY DYNAMIC MODELS

Example / definition

Dynamic incidence matrix (3)
Example: (packet \( k_i \rightarrow k=1; l=1 \); \( h_1[1]=200 \))

\[ \Delta[1,1] = \begin{pmatrix} 1 & \cdot & \cdot \\ \cdot & 0 & \cdot \\ \cdot & \cdot & 0 \\ a_{k-1} & a_k & a_{k-3} \end{pmatrix} \]

\( \tau_1 = 15 \)

\[ \Delta[1,2] = \begin{pmatrix} 0 & \cdot & \cdot \\ 0 & 1 & \cdot \\ 0 & \cdot & 0 \\ 1 & \cdot & \cdot \end{pmatrix} \]

\( \tau_2 = 30 \)

\[ \Delta[1,3] = \begin{pmatrix} 0 & \cdot & \cdot \\ 0 & 0 & \cdot \\ 0 & \cdot & 0 \\ 1 & \cdot & \cdot \end{pmatrix} \]

\( \tau_3 = 45 \)

\[ \Delta[1,4] = \begin{pmatrix} 0 & \cdot & \cdot \\ 0 & 0 & \cdot \\ 0 & \cdot & 0 \\ 0 & \cdot & \cdot \end{pmatrix} \]

\( \tau_4 = 60 \)

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WITHIN-DAY DYNAMIC MODELS

Example / definition

Dynamic incidence matrix (5)

Example:

$\tau_1 = 15$

$\tau_2 = 30$

Dynamic incidence matrix (6)

Example:

$\tau_3 = 45$

$\tau_4 = 60$

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Example / definition

Dynamic incidence matrix (5)
Static link-path incidence matrix is the sum of all dynamic matrices for any given departure interval if the network is cleaned up:

\[ \Delta = \sum_{j \geq l} \Delta[l, j] \text{ e.g. } \Delta[1,1] + \Delta[1,2] + \Delta[1,3] + \ldots + \Delta[1,7] = \Delta \]

Network is empty at interval 7

Dynamic incidence matrix (6)
Travel time on path \( k \) leaving at interval \( l \) is the sum of the forward travel times of all the links along the path in the interval they are reached

\[ TT_k^f(\tau_l) = \sum_a \sum_j \delta_{ak}(l, j) t_a^f(j) \]
\[ TT_l = \sum_j \Delta l^T(l, j) t(j) \]
WITHIN-DAY DYNAMIC MODELS

Example / definition

Path performances (1)

- TRAVEL TIME VARIABLES

\[ \text{TT}_k^f(\tau_j) \]

“forward” travel time on path \( k \): travel time on path \( k \) departing from origin at time \( \tau_j \)

\[
\begin{align*}
\text{TT}_1^f(\tau_1) & = 2273 \text{ [sec]} \\
\text{TT}_1^f(\tau_2) & = 2504 \text{ [sec]} \\
\text{TT}_1^f(\tau_3) & = 2758 \text{ [sec]} \\
\text{TT}_1^f(\tau_4) & = 3118 \text{ [sec]} \\
\frac{\sum \text{TT}_1^f}{\text{TT}_1^f} & = 2663 \text{ [sec]} \quad \text{(average forward travel time along path } 1 \text{ over the entire simulation life)} \\
\frac{\sum \text{TT}_1^f \times F_k}{\text{TT}_1^f} & = 2650 \text{ [sec]} \quad \text{(average forward travel time along path } 1 \text{ weighted with path flows over the entire simulation life)}
\end{align*}
\]

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Specification

OVERALL SUPPLY MODEL

PATH PERFORMANCE MODEL
\[ \mathbf{t}_T = \sum_j \Delta^T(l, j) \cdot t(j) \quad \Delta(l, j) = \Gamma[t(l), . . . , t(j)] \]

LINK PERFORMANCE MODEL
\[ t(r) = t(k(r)) \]

NETWORK FLOW PROPAGATION MODEL
\[ u_a[j] = \sum_{l \leq j} \Delta[l, j] \cdot h[l] \]
\[ f[j] = \sum_{l \leq j} \Delta(l, k) \cdot h[l] \]

Set of equations relating path costs to path flows

taken from:

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OUTLINES

• SUPPLY MODELS
  ✓ Classification factors

• STATIC NETWORK MODELS
  ✓ example/definition
  ✓ specification
  ✓ concluding remarks

• WITHIN-DAY DYNAMIC NETWORK MODELS
  ✓ classification
  ✓ example/definition
  ✓ specification

• STATIC vs. DYNAMIC MODELS

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STATIC vs. DYNAMIC MODELS

Link performances

STATIC SUPPLY MODEL

![Graph showing link travel times over simulation time](image-url)
STATIC vs. DYNAMIC MODELS

Link performances

DYNAMIC SUPPLY MODEL

![Link Travel Times](image)

Simulation Time [min]

Travel Time [min]

0,0

5,0

10,0

15,0

20,0

25,0

0

15

30

45

60

75

90

105

120

a1

-a2

-a3

-a4

-a5

STATIC vs. DYNAMIC MODELS

Trajectory of units of flow

STATIC SUPPLY MODEL

![Units of flow trajectories along path 1](image)

Time [min]

Space [km]

0

10

20

30

40

0

15

30

45

60

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67

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68
STATIC vs. DYNAMIC MODELS

Trajectory of units of flow

DYNAMIC SUPPLY MODEL

Packet trajectories along path 1

STATIC vs. DYNAMIC MODELS

Path performances

Path Travel Times along path 1

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## STATIC vs DYNAMIC MODELS

### Concluding remarks

<table>
<thead>
<tr>
<th>ADVANTAGES</th>
<th>STATIC MODELS</th>
<th>DYNAMIC MODELS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low computational times</td>
<td>Simulation of transients and of over-saturation periods</td>
</tr>
<tr>
<td></td>
<td>Theoretical properties (e.g. existence, uniqueness)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Convergence of assignment models</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DRAWBACKS</th>
<th>STATIC MODELS</th>
<th>DYNAMIC MODELS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Steady state assumption ➔ do not simulate queues</td>
<td>Network flow propagation model ➔ difficult to calibrate</td>
</tr>
<tr>
<td></td>
<td>Underestimation of total travel time under congestion</td>
<td>Time-consuming for large networks</td>
</tr>
<tr>
<td></td>
<td>Memory-less models ➔ simulation day as a succession of different steady states</td>
<td>Theoretical properties uncertain</td>
</tr>
</tbody>
</table>