Traffic Assignment I: Framework for Demand/Performance Interactions

Ennio Cascetta

Modeling and Simulation of Transportation Networks

July 29, 2015

OUTLINE

• INTRODUCTION

• SUPPLY MODELS

• DEMAND MODELS

• ASSIGNMENT MODELS (demand-performances interaction models) see Cascetta’s lecture “Traffic Assignment II: equilibrium and day-to-day dynamic models”
INTRODUCTION
Assignment Models

SIMULATE THE WAY IN WHICH DEMAND AND PERFORMANCES INTERACT IN TRANSPORTATION SYSTEMS, RESULTING FLOWS AND PERFORMANCES ON NETWORK ELEMENTS.
INTRODUCTION

Demand Performances Interaction Models (Assignment)

\[ d = f_1 + f_2 \]

\[ c_1 = c(f_1) \text{ (Congestion)} \]

\[ f_1 = d^*P_1(c_1, c_2) \text{ (Propagation)} \]

In economics

INTRODUCTION

Fields of application of assignment models

- Assignment models as estimators of the present state of the transportation system

- Assignment models for the estimation of present travel demand (see Cascetta's lecture “Calibration and Validation I: Estimation of Origin to Destination Flows from Counts”)

- Assignment models for simulating the effects of changes to the transportation system (e.g. regulations, control and info systems, pricing, infrastructures)

- Assignment models for simulating the effects of land-use changes
INTRODUCTION
Fields of application of assignment models

• Assignment models as estimators of the present state of the transportation system

The results of assignment models can complement direct observations such as link flow counts or path travel time measurements, usually not available for all elements of the system.

INTRODUCTION
Fields of application of assignment models

• Assignment models for the estimation of present travel demand

See Cascetta’s Lecture
Calibration and Validation I: Estimation of Origin to Destination flows from traffic counts
INTRODUCTION

Fields of application of assignment models

• Assignment models for simulating the effects of modifications to the transportation system

Elastic assignment model

- Forecast OD matrices
- Design networks
- Policies (infrastructures and services)

Assignment Model

- Forecast link and path flows
- Link and path travel times and other costs, externalities

INTRODUCTION

Fields of application of assignment models

• Assignment models for simulating the effects of land-use changes

Elastic assignment model

- Land-use changes
- Forecast OD matrices
- Future network

Assignment Model

- Forecast link and path flows
- Link and path travel times and other costs, externalities

Transport - Land use interaction models

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SUPPLY MODELS

Set of equations relating path costs to path flows
SUPPLY MODELS

Classification factors

• TYPES OF TRANSPORTATION SERVICES
  - continuous
  - non continuous (scheduled)

• WITHIN-DAY TEMPORAL DIMENSION
  - static
  - dynamic

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DEMAND MODELS

Equations relating travel demand flows with relevant characteristics \([k_1, k_2, \ldots, k_n]\) to socio-economic variables and supply variables (path costs)

\[ d[k_1, k_2, \ldots, k_n] = d(SE, LoS) \]


DEMAND MODELS

**Travel demand - Choice dimension**

Travel demand is the result of travelers’ choice behavior over several dimensions, e.g.

- Outer (w.r.t. assignment)
  - activity participation
  - tour/trip frequency
  - activity time
  - destination
  - mode/service
- Inner (w.r.t. assignment)
  - departure time
  - path choice/adjustment
  - drivers behavior

DISCRETE CHOICE (RANDOM UTILITY) MODELS TO SIMULATE TRAVELERS’ BEHAVIOR
DEMAND MODELS

Demand flows

• \( d_i \) demand flow between O-D pair \( i \)

• users' classes
users are grouped in segments with a common trip purpose, income, etc.
  ✓ aggregate demand models
  ✓ disaggregate demand models

DEMAND MODELS

Classification factors

PATH CHOICE MODELS
Deterministic vs. Probabilistic

DEMAND ELASTICITY
Rigid vs. Elastic

PATH CHOICE BEHAVIOR
Fully Pre-trip vs. Pre-trip/En-route

INFORMATION AVAILABILITY
Individual experience / information systems (Source)
Pre-trip / En-route (availability time)
Historical / Current / Predictive (Type)

WITHIN-DAY VARIABILITY
Static vs. Dynamic

DAY-TO-DAY VARIABILITY
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DEMAND MODELS

Path choice models

DETERMINISTIC
Users choose only minimum cost alternatives

PROBABILISTIC
Users may choose any available alternative with a probability depending on costs
DEMAND MODELS
*Path choice models*

DETERMINISTIC vs PROBABILISTIC path choice models

**Example**

**NETWORK**

\[ d_{od} = 1000 \text{ veh/h} \]

*path 1:* \[ t_1 = 10 \text{ min} \]

*path 2:* \[ t_2 = 15 \text{ min} \]

**DETERMINISTIC**

\[ V_1 = -\alpha t_1 \text{ prob}[1] = 0.70 \]

\[ V_2 = -\alpha t_2 \text{ prob}[2] = 0.30 \]

\[ \alpha = -5.90 \]

**STOCHASTIC**

\[ F_1 = \text{prob}[1] d_{od} \]

\[ F_2 = \text{prob}[2] d_{od} \]

**Scenario:**

*path 2:* \[ t_2 = 15 \text{ min} \rightarrow 8 \text{ min} \]

**DETERMINISTIC**

\[ V_1 = -\alpha t_1 \text{ prob}[1] = 0.42 \]

\[ V_2 = -\alpha t_2 \text{ prob}[2] = 0.58 \]

\[ \alpha = 5.90 \]
DEMAND MODELS

Path choice models

DETERMINISTIC MODELS

ADVANTAGES
• computational (within-day static)

DISADVANTAGES
• lack of realism
• path flow map is a multi-valued application

PROBABILISTIC MODELS

ADVANTAGES
• more realistic
• wider modeling flexibility
• path flow map is a continuous function

DISADVANTAGES
• computational (within-day static), (increasingly less relevant)

The simulation of ATIS strategies affecting users’ perception requires probabilistic models

DEMAND MODELS

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Demand elasticity

The level of demand results from outer choice dimensions, such as destination, mode, etc.

RIGID
The demand on outer choice dimensions does not depend on cost variations, e.g. mode choice is rigid w.r.t. travel time changes

ELASTIC
The demand on outer choice dimensions depends on costs

---

RIGID vs ELASTIC DEMAND

Example

\[ d_{od} = 1300 \text{ pax/h} \]

<table>
<thead>
<tr>
<th>Mode</th>
<th>Path 1</th>
<th>Path 2</th>
<th>Transit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car</td>
<td>10 min</td>
<td>15 min</td>
<td>20 min</td>
</tr>
<tr>
<td>Transit</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ V(\text{car}) = -\alpha_1 t^{\min_{\text{car}}} = -\alpha_1 t_{\min(c_1, c_2)} = -\alpha_1 t_{c_1} \]

\[ V(\text{transit}) = -\alpha_2 t_t \]

\[ \alpha_1 = 0.052; \quad \alpha_2 = 0.096 \]

Hp: deterministic path choice model
DEMAND MODELS

Demand elasticity

RIGID vs ELASTIC DEMAND

Example (cont’d)

Scenario:

Car (path 2): \( t_2 = 15 \text{ min} \rightarrow 8 \text{ min} \)

RIGID DEMAND

\[
V(\text{car}) = -\alpha_1 t_{\text{min}}^\text{car} = -\alpha_1 \min\{t_{c1}, t_{c2}\} = -\alpha_1 t_{c1}
\]

\[
V(\text{transit}) = -\alpha_2 t_t
\]

\[
\alpha_1 = 0.052; \quad \alpha_2 = 0.096
\]

ELASTIC DEMAND

\[
p[\text{car}] = 0.80
\]

\[
p[\text{transit}] = 0.20
\]

DEMAND MODELS

Demand elasticity

RIGID DEMAND

FEATURES

• simpler models
• lack of realism if l.o.s. attributes change significantly

ELASTIC DEMAND

FEATURES

• more complex models and algorithms
• more realism

The simulation of the effects of policies and control systems affecting choice dimensions other than path (and departure time) requires elastic demand models
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Path choice behaviour

FULLY PRE-TRIP
Users choose the path before starting their trip based on available information

PRE-TRIP / EN-ROUTE
Users choose a path or a set of paths based on available information before starting their trip and adjust their choices during the trip reacting to new information
DEMAND MODELS
Path choice behaviour

FULLY PRE-TRIP vs PRE-TRIP/EN-ROUTE path choice behaviour

Example

\[ V_1 = -\alpha t_1 \]
\[ V_2 = -\alpha t_2 \]
\[ V_3 = -\alpha t_2 \]
\[ \alpha = 4 \]

\[ d_{tot} = 1000 \text{ veic/h} \]

\[ \text{Path 1} = 9 \text{ min} \]
\[ \text{Path 2a} = 10 \text{ min} \]
\[ \text{Path 2b} = 10 \text{ min} \]

VMS: providing information on queues on links exiting node B

FULLY PRE-TRIP

No queue

\[ F_1 = 400 \text{ veh/h} \]
\[ F_{2a} = 300 \text{ veh/h} \]
\[ F_{2b} = 300 \text{ veh/h} \]

PRE-TRIP/EN ROUTE

\[ \text{prob}[1] = 0.40 \]
\[ \text{prob}[2] = 0.30 \]
\[ \text{prob}[3] = 0.30 \]

Demand Models
Path choice behaviour

FULLY PRE-TRIP vs PRE-TRIP/EN-ROUTE path choice behaviour

Example (cont’d)

Queue on link BC belonging to path 2a (VMS compliance: 50%)

FULLY PRE-TRIP

\[ F_1 = 400 \text{ veh/h} \]
\[ F_{2a} = 300 \text{ veh/h} \]
\[ F_{2b} = 300 \text{ veh/h} \]

PRE-TRIP/EN ROUTE

\[ F_1 = 400 \text{ veh/h} \]
\[ F_{2a} = 150 \text{ veh/h} \]
\[ F_{2b} = 450 \text{ veh/h} \]
DEMAND MODELS

Path choice behaviour

FULLY PRE-TRIP

ADVANTAGES
• simpler mathematical formulation of choice alternatives (paths) and selection models

DRAWBACKS
• lack of realism in simulating unreliable systems and/or systems with additional information provided or acquired by users during the trip

PRE-TRIP/EN-ROUTE

ADVANTAGES
• more realism in representing actual behavior

DRAWBACKS
• more complex mathematical models

The simulation of systems in which significant information is not available before departure (e.g. Unreliable services and/or en route ATIS) requires mixed pre-trip/en-route path choice behavior

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DEMAND MODELS

Information availability – Information sources

INDIVIDUAL EXPERIENCE
Information obtained through personal or shared experience

INFORMATION SYSTEM
• DESCRIPTIVE INFORMATION - description of network affects
  ✓ CHOICE SET ADJUSTMENT
  ✓ ATTRIBUTES PERCEPTION ADJUSTMENT

• PRESCRIPTIVE INFORMATION - recommendation on what to do
  ✓ COMPLIANCE MODELS

DEMAND MODELS

Information availability – Time of availability

PRE-TRIP
Information available prior to trip departure
CHOICE DIMENSION AFFECTED: Mode, Departure time, Path, (Destination, trip-frequency, tour type, activity schedule, etc.)

EN-ROUTE
Information available during the trip
CHOICE DIMENSION AFFECTED: Path

Examples

<table>
<thead>
<tr>
<th>INDIVIDUAL EXPERIENCE</th>
<th>PRE-TRIP</th>
<th>EN-ROUTE</th>
</tr>
</thead>
<tbody>
<tr>
<td>INFO. SYSTEM</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Descriptive</td>
<td>Previous experience, Word of mouth</td>
<td>Queue ahead, Bus arrival at stop</td>
</tr>
<tr>
<td>Prescriptive</td>
<td>Radio, Internet, Television</td>
<td>Variable Message Signs (e.g. “queue ahead”), In-vehicle Navigation systems</td>
</tr>
<tr>
<td></td>
<td>Radio, Internet, Television</td>
<td>Variable Message Signs (e.g. “turn left”), In-vehicle Navigation systems</td>
</tr>
</tbody>
</table>
DEMAND MODELS

Information availability – Type of information

HISTORICAL INFORMATION
Concerns the state of the network during the previous days

CURRENT INFORMATION
Based on the current state of the network

PREDICTIVE INFORMATION
Concerns the future conditions on the network

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DEMAND MODELS

Within day variability

• WITHIN-DAY (INTRA-PERIOD) STATIC

• WITHIN-DAY (INTRA-PERIOD) DYNAMIC
  Departure Time Choice

DEMAND MODELS

Within day variability

WITHIN-DAY STATIC DEMAND MODELS

\( h_k \) flow on path \( k \)

\( h = [h_k] \) total flow vector

\( g_k \) cost on path \( k \)

\( g = [g_k] \) path systematic utility vector

\( TT_k \) travel time on path \( k \)

\( TT = [TT_k] \) travel time vector

\( p_k \) probability of choosing path \( k \)

\( P = [p_k]_{k,od} \) path probability matrix

\( g_k = \beta_1 \cdot TT_k + \beta_2 \cdot \text{monetary cost} + \ldots. \)

\( p_k = p_k(g) \)

\( h_k = d_{od} \cdot p_k \quad k \in K_{od} \)

\( h = P(g) \cdot d \)

(rigid demand – pre-trip path choice example)
WITHIN-DAY STATIC DEMAND MODELS
Example of Path probability matrix \( P \)

\[
\begin{align*}
G & = (N, L) \\
N & = \{(1, 2, 3, 4)\} \\
L & = \{(1,2), (1,3), (2,3), (2,4), (3,4)\}
\end{align*}
\]

**DEMAND MODELS**

*Within day variability*

**WITHIN-DAY STATIC DEMAND MODELS**

Example of Path probability matrix \( P \) (cont’d)

\[
g^T = [6 \ 4 \ 2 \ 4 \ 2 \ 1] \\
p_{o,d} = \frac{\exp(-g_{o,d}/\theta)}{\sum d_{o,d} \exp(-g_{o,d}/\theta)}; \quad \theta = 2
\]

\[
P_{14} = \begin{bmatrix} 0.090 \\ 0.245 \\ 0.665 \end{bmatrix}; \quad P_{24} = \begin{bmatrix} 0.269 \\ 0.731 \end{bmatrix}; \quad P_{34} = [1.000]
\]

<table>
<thead>
<tr>
<th>Path</th>
<th>O-D</th>
<th>1-4</th>
<th>2-4</th>
<th>3-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.090</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.245</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0.665</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0.269</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0.731</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
<td>1.000</td>
<td>0</td>
</tr>
</tbody>
</table>

\[
h = \begin{bmatrix} 90 \\ 245 \\ 665 \\ 404 \\ 1097 \\ 800 \end{bmatrix} \\
P = \begin{bmatrix} 0.090 & 0 & 0 \[ \end{bmatrix} \\
d = \begin{bmatrix} 1000 \[ \end{bmatrix} \\
1500 \[ 800 \]
\]

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DEMAND MODELS

Within day variability

WITHIN-DAY DYNAMIC DEMAND MODELS

Path & departure time choice model

\( h_{kj} \) flow on path \( k \), leaving during interval \( j \)

\( TT_{kj} \) travel time on path \( k \) leaving during interval \( j \)

\( ELAD_{kj} \) early/late arrival systematic disutility of using path \( k \) and leaving during interval \( j \)

\( g_{kj} \) cost on path \( k \), leaving during interval \( j \)

\( g = [g_{kj}] \) path cost vector

\( p_{kj} \) probability of choosing path \( k \) and leaving during interval \( j \)

\( p_j \) path probability vector leaving during interval \( j \)

\[ g_{kj} = \beta_1 \cdot TT_{kj} + \beta_2 \cdot ELAD_{kj}(TT_{kj}) + \ldots \]

\[ p_{kj} = p_k(g) \]

\[ h_{kj} = d_{od} \cdot p_{kj}(g) \quad k \in K_{od} \]

\[ h = P(g) \cdot d \]

(rigid demand – pre-trip departure and path choice - discrete flow example)

DEMAND MODELS

Within day variability

Example of departure time/path choice model

\( d(O,D) = d(1,4) = 1000 \text{ veic/h} \) (desired arrival time = 8:45)

\( D = (1,2,3,4) \)

\( G = (N, L) \)

\( N = (1, 2, 3, 4) \)

\( L = (1, (2,3), (2,3), (4,5), (3,4)) \)

\( (Logit \ model) \)

\[ p_{kj} = \frac{\exp(-TT_{kj} - 1.5 \cdot ELAD_{kj})/\theta)}{\sum_{k \in K_{od}} \exp(-TT_{kj} - 1.5 \cdot ELAD_{kj})/\theta)} \]

\( \theta = 20 \)
**DEMAND MODELS**

*Within day variability*

Example of departure time /path choice model

d(O,D) = d(1,4) = 1000 veic/h (desired arrival time = 8:45)

<table>
<thead>
<tr>
<th>Path</th>
<th>Cost</th>
<th>Time</th>
<th>Arrival</th>
<th>ELAD</th>
<th>$g(kj)$</th>
<th>$exp(g(kj))$</th>
<th>$P(kj)$</th>
<th>$h(kj)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interval 1 (departing at 7:45)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>k1</td>
<td>70</td>
<td>8.55</td>
<td>10 late arrival</td>
<td>-4.25</td>
<td>0.014</td>
<td>4%</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>k2</td>
<td>45</td>
<td>8.30</td>
<td>15 early arrival</td>
<td>-3.38</td>
<td>0.034</td>
<td>9%</td>
<td>95</td>
<td></td>
</tr>
<tr>
<td>k3</td>
<td>25</td>
<td>8.10</td>
<td>35 early arrival</td>
<td>-3.88</td>
<td>0.021</td>
<td>6%</td>
<td>58</td>
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<tr>
<td>Interval 2 (departing at 8:15)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>k1</td>
<td>50</td>
<td>9.05</td>
<td>20 late arrival</td>
<td>-4.00</td>
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<td>5%</td>
<td>51</td>
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<tr>
<td>k2</td>
<td>35</td>
<td>8.50</td>
<td>5 late arrival</td>
<td>-2.13</td>
<td>0.119</td>
<td>33%</td>
<td>331</td>
<td></td>
</tr>
<tr>
<td>k3</td>
<td>15</td>
<td>8.30</td>
<td>15 early arrival</td>
<td>-1.88</td>
<td>0.153</td>
<td>43%</td>
<td>426</td>
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Individual experiences / information systems (Source)
Pre-trip / En-route (Availability time)
Historical / Current / Predictive (Type)

**WITHIN-DAY VARIABILITY**
Static vs. Dynamic

**DAY-TO-DAY VARIABILITY**
Cost updating model
Information combination model
Choice updating model
DEMAND MODELS

Classification factors

DAY-TO-DAY VARIABILITY

• Modeling past experience (learning) ➔ Cost updating model

• Modeling (pre-trip /en-route) combination of different sources of Information ➔ Information combination model

• Modeling choice updating (including inertia) ➔ Choice updating model

COST UPDATING MODEL

Gives the expected cost at day t based on previous experience at day t-1

\[ g_{\text{exp}}^t = g_1(g_{\text{pre}}^{t-1}, g_{\text{act}}^{t-1}) \]

Example:

INDIVIDUAL vs. COLLECTIVE UPDATING
DEMAND MODELS
Day-to-day dynamic demand models

COST UPDATING MODEL
Path based models

• Exponential smoothing filter

\[ g_{\text{exp}}^t = \beta g_{\text{act}}^{t-1} + (1-\beta) g_{\text{pre}}^{t-1} \]

\( \beta \in (0,1] \) the weight given to costs actually experienced

• Moving average filter

• Bayesian updating filter

COST UPDATING MODEL
Link based models

\( c_{\text{exp}}^t \) expected link cost vector at day \( t \)
\( c_{\text{pre}}^{t-1} \) pre-trip link cost vector at day \( t-1 \)
\( c_{\text{act}}^{t-1} \) actual link cost vector at day \( t-1 \)

\[ c_{\text{exp}}^t = c(c_{\text{pre}}^{t-1}, c_{\text{act}}^{t-1}) \]
\[ g_{\text{exp}}^t = \Delta c_{\text{exp}}^t \]

• Exponential smoothing filter

\[ c_{\text{exp}}^t = \beta c_{\text{act}}^{t-1} + (1-\beta)(c_{\text{pre}}^{t-1}) \]

link and path based models may lead to different results for non-linear filters and non-additive path costs
DEMAND MODELS
Day-to-day dynamic demand models

COST UPDATING MODEL - EXAMPLE

\[ g_{\text{exp}}^t = 0.3 \cdot g_{\text{act}}^{t-1} + 0.7 \cdot g_{\text{pre}}^{t-1} \]

\[
\begin{bmatrix}
45.0 \\
45.0 \\
45.0 \\
71.3
\end{bmatrix}
\]

\[
\begin{bmatrix}
46.3 \\
46.3
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.3 \cdot 71.3 + 0.7 \cdot 45.0 \\
0.3 \cdot 46.3 + 0.7 \cdot 45.0
\end{bmatrix}
\]

\[
\begin{bmatrix}
52.9 \\
45.0
\end{bmatrix}
\]

DEMAND MODELS
Day-to-day dynamic demand models

INFORMATION COMBINATION MODEL
Gives the pre-trip (combined) path cost at day \( t \) based on expected and “information system” path costs at day \( t \)

\[
g_{\text{pre}}^t = g_2(g_{\text{exp}}^t, g_{\text{info}}^t)
\]

Example

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DEMAND MODELS

Day-to-day dynamic demand models

INFORMATION COMBINATION MODEL

Path based models

• Exponential smoothing filter

\[ g_{\text{pre}}^t = \lambda_i g_{\text{info}}^t + (1 - \lambda_i) g_{\text{exp}}^t \]

\( \lambda_i \in (0,1] \) the weight given to information system forecasted today costs (i.e. measure of system credibility)

• Moving average filter

• Bayesian updating filter

DEMAND MODELS

Day-to-day dynamic demand models

INFORMATION COMBINATION MODEL

Link based models

\( c_{\text{pre}}^t \) pre-trip link cost vector at day \( t \)
\( c_{\text{exp}}^t \) expected link cost vector at day \( t \)
\( c_{\text{info}}^t \) information system link cost vector at day \( t \)

\[ c_{\text{pre}}^t = \lambda_i (c_{\text{exp}}^{t-1}, c_{\text{info}}^t) \]
\[ g_{\text{pre}}^t = \Delta^t c_{\text{pre}}^t \]

• Exponential smoothing filter

\[ c_{\text{pre}}^t = \lambda_i c_{\text{info}}^{t-1} + (1 - \lambda_i) c_{\text{exp}}^t \]

link and path based model generally lead to different results
DEMAND MODELS

Day-to-day dynamic demand models

INFORMATION COMBINATION MODEL

EXAMPLE

\[
\begin{align*}
\mathbf{c}_{21, \text{exp}} &= 15 \\
\mathbf{c}_{24, \text{exp}} &= 30 \\
\mathbf{c}_{33, \text{exp}} &= 15 \\
\mathbf{c}_{52, \text{exp}} &= 15
\end{align*}
\]

\[
\mathbf{g}_{\text{pre}}^{t-1} = \begin{bmatrix} 45.0 \\ 45.0 \\ 45.0 \end{bmatrix}, \quad \mathbf{g}_{\text{exp}}^{t-1} = \begin{bmatrix} 15 + 15 + 1.5 \cdot 15 \\ 30 + 1.5 \cdot 15 \\ 15 + 30 \end{bmatrix} = \begin{bmatrix} 52.5 \\ 52.5 \\ 45.0 \end{bmatrix}
\]

\[
\mathbf{g}_{\text{info}}^{t-1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \mathbf{g}_{\text{exp}}^{t} = \begin{bmatrix} 45.0 \\ 45.0 \\ 45.0 \end{bmatrix}
\]

\[
\mathbf{g}_{\text{pre}}^{t} = 0.2 \cdot \mathbf{g}_{\text{info}}^{t} + 0.8 \cdot \mathbf{g}_{\text{exp}}^{t}
\]

DEMAND MODELS

Day-to-day dynamic demand models

INFORMATION SYSTEM MODEL

Information strategies

Daily Adjustment
\[
\mathbf{g}_{\text{info}}^{t} = \rho \mathbf{g}_{\text{pre}}^{t} + (1 - \rho) \mathbf{g}_{\text{info}}^{t-1}
\]

Fixed Info
\[
\mathbf{g}_{\text{info}}^{t} = \mathbf{g}_{\text{info}}^{t-0}
\]

Delayed Info
\[
\mathbf{g}_{\text{info}}^{t} = \mathbf{g}_{\text{pre}}^{t} \quad \text{i.e. no information learning}
\]

Fully-accurate Information
\[
\mathbf{g}_{\text{info}}^{t} = \mathbf{g}_{\text{actual}}^{t}
\]

**DEMAND MODELS**

*Day-to-day dynamic demand models*

**CHOICE UPDATING MODEL**

*On day t*

- \( p_{k/k'} \): probability of choosing route \( k \), conditional to route \( k' \) chosen at day \( t-1 \)
- \( P_t \): \( [p_{k/k'}]_{k,k'} \) route choice conditional probability matrix

Example:

\[ h_t = \sum_{k'} p_{k/k'} h_{t-1} \]

**EXPOSURE SMOOTHING MODEL**

\[ h_t = \alpha P_t h_{t-1} + (1-\alpha) I \]

where:

- \( \alpha \in [0,1] \) the probability of reconsidering day \( t-1 \) choices
- \( P_t \) the path choice probability matrix conditional to reconsidering yesterday choice

**THRESHOLD SWITCHING MODEL**

Switching depends on the difference between expected and experienced costs w.r.t. a deterministic or stochastic threshold

\[ \alpha = \alpha(g_{\text{pre}} g_{\text{act}}) \]

**TRANSITION COST MODELS**

Extra-costs for changing yesterday choice

\[ p_{k,k'} = \frac{\exp(g_{\text{pre,k}} + T_{C_{k',k}})}{\sum_{k} \exp(g_{\text{pre,h}} + T_{C_{h,k}})} \]

\( T_{C_{k',k}} \) transition cost from path \( k' \) at day \( t-1 \) to path \( k \) at day \( t \)
DEMAND MODELS
Day-to-day dynamic demand models

CHOICE UPDATING MODEL
Compliance with information

\[ \alpha_t = \mu m(\text{Inaccuracy}_t^{-1}, \text{MarketPenetration}) + (1-\mu) \alpha_{t-1} \]

- \( m(\text{Inaccuracy}_t^{-1}, \text{MarketPenetration}) \):
  - depends on market penetration
  - Inaccuracy (unreliability) increases \( \rightarrow \) compliance decreases
  - Linear model (approximated)
    - more sophisticated = not explicit (Ben-Elia et al. 2014)

\[ \| g^{t}_{\text{info}} - g^{t}_{\text{actual}} \| \]

DEMAND MODELS
Day-to-day dynamic demand models

CHOICE UPDATING MODEL - EXAMPLE

\[ d(1,4) = 1500 \text{ veic/h} \]
\[ t - 1 \]
\[ t \]
\[ h' = 0.1^*P \left[-\frac{1}{60}g_{pre} \right] d + 0.9^*h^{-1} \]

\[
\begin{bmatrix}
500 \\
500 \\
500
\end{bmatrix}
\]

\[
\begin{bmatrix}
31.9 \\
24.4 \\
24.4
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.31 \\
0.35 \\
0.35
\end{bmatrix}
\]

\[
\begin{bmatrix}
460 \\
520 \\
520
\end{bmatrix}
\]

\[
\begin{bmatrix}
496 \\
502 \\
502
\end{bmatrix}
\]

\[
0.10 \quad 0.90
\]

Example

TEST NETWORK

SHORTEST PATH
between origin O and destination D
"free-flow" travel time = 1680 sec
"actual" (equilibrium) travel time = 1870 sec
DEMAND MODELS

Day-to-day dynamic demand models

Example

<table>
<thead>
<tr>
<th>Users’ classes</th>
<th>informed users (I)</th>
<th>Not informed users (NI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Habitual users (A)</td>
<td>IA</td>
<td>NIA</td>
</tr>
<tr>
<td>Not-Habitual users (NA)</td>
<td>INA</td>
<td>NINA</td>
</tr>
</tbody>
</table>

Cost updating and information acquisition models per user class

• Informed-Habitual users (IA)

\[ g_{pre}^{IA} = \lambda^{IA} \cdot g_{info}^I + (1 - \lambda^{IA}) \cdot \left[ \beta^{IA} \cdot g_{act}^{I-1} + (1 - \beta^{IA}) \cdot g_{pre}^{IA} \right] \quad \forall t \]

• Informed-Not Habitual users (INA)

\[ g_{pre}^{INA} = g_{info}^I \quad \forall t \]

• Not Informed-Habitual users (NIA)

\[ g_{pre}^{NIA} = \beta^{NIA} \cdot g_{act}^{I-1} + (1 - \beta^{NIA}) \cdot g_{pre}^{NIA} \quad \forall t \]

\[ g_{NIA}^I = g_{NIA}^{I-1} \]

• Not Informed- Not Habitual users (NINA)

\[ g_{pre}^{NINA} = g_{pre}^{I-1,NINA} = g_0^I \quad \forall t \]

being \textbf{g} the vector of the free-flow path travel times

DEMAND MODELS

Day-to-day dynamic demand models

Example

Percentage distribution of the demand flow among users’ classes and parameters of the day-to-day path choice models

<table>
<thead>
<tr>
<th></th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \lambda )</th>
<th>% on total OD demand flow</th>
</tr>
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<tbody>
<tr>
<td>IA</td>
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<td>0.2</td>
<td>0.7</td>
<td>15%*</td>
</tr>
<tr>
<td>INA</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>15%*</td>
</tr>
<tr>
<td>NIA</td>
<td>0.2</td>
<td>0.2</td>
<td>0</td>
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<td>NINA</td>
<td>1</td>
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100%

*market penetration of the ATIS = 30% (IA+INA)
DEMAND MODELS

*Day-to-day dynamic demand models*

**Analysis of path travel time day-to-day evolution**

- Recurrent Congestion (= actual travel time at equilibrium, on the minimum-cost path, equal to 1870 sec)
  
  informed vs. non-informed users

- Non-Recurrent Congestion (= accident occurring at day 15 causing an increase of travel time on the minimum-cost path up to 2000 sec)
  
  informed vs. non-informed users

---

**DEMAND MODELS**

*Day-to-day dynamic demand models*

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**PRE-TRIP AND ACTUAL COST ON THE SHORTEST PATH**

**Recurrent congestion INFORMED USERS**
DEMAND MODELS

Day-to-day dynamic demand models

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PRE-TRIP AND ACTUAL COST ON THE SHORTEST PATH

Recurrent congestion
NON-INFORMED USERS

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DEMAND MODELS

Day-to-day dynamic demand models

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PRE-TRIP AND ACTUAL COST ON THE SHORTEST PATH

Non-recurrent congestion (i.e. accident at day 15)
INFORMED USERS

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DEMAND MODELS

Day-to-day dynamic demand models

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PRE-TRIP AND ACTUAL COST ON THE SHORTEST PATH

Non-recurrent congestion (i.e. accident at day 15)

NON-INFORMED USERS

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