Traffic Assignment II: Equilibrium and day-to-day dynamics

Ennio Cascetta

Modeling and Simulation of Transportation Networks

July 29, 2015

OUTLINE

• INTRODUCTION

• SUPPLY MODELS

• DEMAND MODELS

• SUPPLY/DEMAND INTERACTION MODELS (Assignment models)
ASSIGNMENT MODELS

SIMULATE DEMAND-SUPPLY INTERACTIONS, RESULTING FLOWS AND PERFORMANCES ON NETWORK ELEMENTS.

ASSIGNMENT MODELS

Classification factors

- SUPPLY MODELS
  - WITHIN-DAY STATIC
  - WITHIN-DAY DYNAMIC
  - ...

- DEMAND MODELS
  - WITHIN-DAY STATIC
  - WITHIN-DAY DYNAMIC
  - ...

- ASSIGNMENT MODELS
  - Not Congested Systems
  - Congested Systems
    - Equilibrium
    - Dynamic Processes
ASSIGNMENT MODELS

Classification factors

- **SUPPLY MODELS**
  - WITHIN-DAY STATIC
  - WITHIN-DAY DYNAMIC
  - ...

- **DEMAND MODELS**
  - WITHIN-DAY STATIC
  - WITHIN-DAY DYNAMIC
  - ...

- **ASSIGNMENT MODELS**
  - Not Congested Systems
  - Congested Systems
    - Equilibrium
    - Dynamic Processes

ASSIGNMENT MODELS

**Congestion**

Occurs in most transportation systems, generally when multiple users interact with each other.

Worsening the overall performances, such as the mean speed or the travel time, since a vehicle may not be able to move at the desired speed.

Example: motorway links (see Lecture: Traffic Performance IV)

\[ tr(f_a) = \frac{L}{v_{0l}} + \gamma \left( \frac{L}{v_{cl}} - \frac{L}{v_{pl}} \right) \left( \frac{f_a}{Q_a} \right)^{\gamma_1} \]

where:

- \( L \) is the length of link \( l \);
- \( v_{0l} \) is the free flow average speed;
- \( v_{pl} \) is the average speed with flow equal to capacity;
- \( Q_a \) is the link capacity, i.e. the average maximum number of equivalent vehicles that can travel along the road section in a time unit. Capacity is usually obtained as the product of the number of lanes on the link \( l \), \( N_l \), and lane capacity, \( Q_{ul} \);
- \( \gamma \) are parameters of the function.
ASSIGNMENT MODELS
Not congested network

DEMAND MODEL
- OD demand
- Path/Departure time choice model

SUPPLY MODEL
- Path Flows (h)
- Path cost
- Network Flow Propagation Model
- Link Flows (f)
- Link costs (c)
- Link Performance Model

ASSIGNMENT MODELS
Not congested network

WITHIN-DAY STATIC
Supply
\[ c = c^0 \]
\[ g = \Delta^T c \]
\[ f = \Delta h \]

Demand
\[ h = P(g) \, d \]
\[ f = \Delta P(\Delta^T c^0) \, d \]

WITHIN-DAY DYNAMIC
Supply
\[ t_j = t_j^0 \]
\[ TT_j = \sum_{m \in j} \Delta^T(j, m) \, t_m^0 \]
\[ g_j = TT_j + \text{ELAD}_j(TT_j) \]

Demand
\[ h_j = P_j(g) \, d \]
\[ f_j = \sum_{l \in j} \Delta(l, j) \ast h(l) \]
\[ \Delta_{lj} = \Gamma(t_l^0, \ldots, t_j^0) \]

\[ f_j = \sum_{l \in j} \Delta_{lj} \, P_l(g) \, d \]
\[ g_j = f(t^0) + \text{ELAD}_j(f(t^0)) \]

taken from:
ASSIGNMENT MODELS

Acronyms for uncongested traffic assignment models in the literature

- **DNL**  Deterministic Network Loading or “All or Nothing” (AoN)
- **SNL**  Stochastic Network Loading
- **DDNL** Dynamic Deterministic Network Loading
- **DSNL** Dynamic Stochastic Network Loading

ASSIGNMENT MODELS

Not congested network

**Example**
Let us consider OD pair 1-4 in the following network

![Diagram of a network with OD pair 1-4 and links 1, 2, 3, 4, and 5.]

**Demand**
d(1,4)=150 veic/h

**Incidence matrix**

![Incidence matrix for the network diagram.]
ASSIGNMENT MODELS
Not congested network

Example

Free flow cost

Path choice model
\[ P(g_k) = \exp\left[-\frac{(1/60) \cdot g_k}{60}\right] \]

Link cost function
\[ c(f) = c_0 + \left(\frac{f}{\text{Cap}}\right)^2 \quad \text{Cap} = 200 \text{ veic/h} \]

Demand
\[ d(1,4) = 150 \text{ veic/h} \]

\[ \begin{array}{|c|c|c|c|c|c|}
\hline
\text{Path #} & g & P(g) & h & Pd & f= Dh \\hline
1 & 5,0 & 15 & 33\% & 50 & a1 \quad 100 \quad 5,3 \\hline
2 & 10,0 & 15 & 33\% & 50 & a2 \quad 50 \quad 10,1 \\hline
3 & 5,0 & 15 & 33\% & 50 & a3 \quad 50 \quad 5,1 \\hline
4 & 10,0 & 3 & 33\% & 50 & a4 \quad 50 \quad 10,1 \\hline
5 & 5,0 & 3 & 33\% & 50 & a5 \quad 100 \quad 5,3 \\hline
\end{array} \]

\[ c(f) \sim c_0 \quad \text{not congested network} \]

ASSIGNMENT MODELS
Classification factors

- SUPPLY MODELS
  - WITHIN-DAY STATIC
  - WITHIN-DAY DYNAMIC

- DEMAND MODELS
  - WITHIN-DAY STATIC
  - WITHIN-DAY DYNAMIC

- ASSIGNMENT MODELS
  - Not Congested Systems
  - Congested Systems
    - Equilibrium
    - Dynamic Processes
### ASSIGNMENT MODELS

**Congested network**

**Example**

**Demand** $d(1,4) = 1500$ veic/h

<table>
<thead>
<tr>
<th>Path #</th>
<th>$g=D^c$</th>
<th>$P(g)$</th>
<th>$h=Pd$</th>
<th>$f=Dh$</th>
<th>$c=c(f)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>5,0</td>
<td>1</td>
<td>15</td>
<td>33%</td>
<td>500</td>
</tr>
<tr>
<td>$a_2$</td>
<td>10,0</td>
<td>2</td>
<td>15</td>
<td>33%</td>
<td>500</td>
</tr>
<tr>
<td>$a_3$</td>
<td>5,0</td>
<td>3</td>
<td>15</td>
<td>33%</td>
<td>500</td>
</tr>
<tr>
<td>$a_4$</td>
<td>10,0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_5$</td>
<td>5,0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$c(f) \neq c_0$

$f_{k-1} \neq f_k$

**DAY TO DAY STATIC** (equilibrium)

Mutually consistent flows and costs

**DAY TO DAY DYNAMIC** (dynamic processes)

Evolution over time of the system state

---

Department of Transportation Engineering
University of Naples "Federico II"
ASSIGNMENT MODELS

Classification factors

• SUPPLY MODELS
  ✓ WITHIN-DAY STATIC
  ✓ WITHIN-DAY DYNAMIC

• DEMAND MODELS
  ✓ WITHIN-DAY STATIC
  ✓ WITHIN-DAY DYNAMIC

• ASSIGNMENT MODELS
  ✓ Not Congested Systems
  ✓ Congested Systems
    ➢ Equilibrium
    ➢ Dynamic Processes

ASSIGNMENT MODELS
Congested network – Equilibrium models
ASSIGNMENT MODELS
Congested network – Equilibrium models

**WITHIN-DAY STATIC**

Supply
\[ c = c(f) \]
\[ g_i = \Delta_i^c \]
\[ f = \Delta h \]

Demand
\[ h = P(g) \, d \]

**WITHIN-DAY DYNAMIC**

\[ t = t(f) \]
\[ \Delta i_j = \Gamma(t_i, ..., t_j) \]
\[ f_j = \Sigma \Delta_{i,j} \, h(l) \]

\[ g_j = TT_j + ELAD_j(TT_j) \]

**FIXED-POINT MODELS**

\[ f = \Delta P(\Delta^c) \, d \]
\[ c = c(f) \]

\[ f^* = \Sigma \Delta_{i,j} [t (f^*)] \, P_j(\Sigma \Delta_{i,j} [t (f^*)] \, t(f^*) + ELAD(\Gamma(t(f^*))) \, d \]

Acronyms for equilibrium assignment models in the literature

**DUE**  Deterministic User Equilibrium

**SUE**  Stochastic User Equilibrium

**DDUE**  Dynamic Deterministic User Equilibrium

**DSUE**  Dynamic Stochastic User Equilibrium
ASSIGNMENT MODELS

Congested network – Equilibrium models

MSA-F A ALGORITHM

Algorithm structure:
• \(k = 0\) (initialization)
• \(c^0 = c(f=0)\)
• \(f^0 = f_{SNL}^0\)

• \(k = k + 1\)
• \(c^k = c(f^{k-1})\)
• \(f^k = (k-1)/k f^{k-1} + (1/k) f_{SNL}^k\)
• convergence criterion check

\[ f_{SNL}^k = f_{SNL}^{k-1} \]

Note: faster convergence reached if \(k\) reset after some iterations

Optimal \(k \in [5-10]\)

ASSIGNMENT MODELS

Congested network – Equilibrium models

MSA-F A ALGORITHM

Example (1/4)

<table>
<thead>
<tr>
<th>(k=0) (init)</th>
<th>Path</th>
<th>(c^0)</th>
<th>(g)</th>
<th>(p)</th>
<th>(h)</th>
<th>(f_{SNL})</th>
<th>(f^0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a1)</td>
<td>5.0</td>
<td>1</td>
<td>15</td>
<td>33%</td>
<td>2667</td>
<td>(a1)</td>
<td>5333</td>
</tr>
<tr>
<td>(a2)</td>
<td>10.0</td>
<td>2</td>
<td>15</td>
<td>33%</td>
<td>2667</td>
<td>(a2)</td>
<td>2667</td>
</tr>
<tr>
<td>(a3)</td>
<td>5.0</td>
<td>3</td>
<td>15</td>
<td>33%</td>
<td>2667</td>
<td>(a3)</td>
<td>2667</td>
</tr>
<tr>
<td>(a4)</td>
<td>10.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(a4)</td>
<td>2667</td>
</tr>
<tr>
<td>(a5)</td>
<td>5.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(a5)</td>
<td>5333</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(k=1) (c(f^k))</th>
<th>Path</th>
<th>(g)</th>
<th>(p)</th>
<th>(h)</th>
<th>(f_{SNL})</th>
<th>(f^k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a1)</td>
<td>716.1</td>
<td>1</td>
<td>1615</td>
<td>0%</td>
<td>0</td>
<td>(a1)</td>
</tr>
<tr>
<td>(a2)</td>
<td>187.8</td>
<td>2</td>
<td>904</td>
<td>50%</td>
<td>4000</td>
<td>(a2)</td>
</tr>
<tr>
<td>(a3)</td>
<td>182.8</td>
<td>3</td>
<td>904</td>
<td>50%</td>
<td>4000</td>
<td>(a3)</td>
</tr>
<tr>
<td>(a4)</td>
<td>187.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(a4)</td>
</tr>
<tr>
<td>(a5)</td>
<td>716.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(a5)</td>
</tr>
</tbody>
</table>

taken from:
ASSIGNMENT MODELS
Congested network – Equilibrium models

MSA-FA ALGORITHM
f^2 = (1/2) f^1 + (1/2) f^{SNL}

Example

<table>
<thead>
<tr>
<th>k=2</th>
<th>c(f^{k-1})</th>
<th>Path #</th>
<th>g</th>
<th>P</th>
<th>h</th>
<th>f^{SNL}</th>
<th>f^k</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>405.0</td>
<td>1</td>
<td>815</td>
<td>32%</td>
<td>2520</td>
<td>a1</td>
<td>5260</td>
</tr>
<tr>
<td>a2</td>
<td>405.0</td>
<td>2</td>
<td>810</td>
<td>34%</td>
<td>2740</td>
<td>a2</td>
<td>2740</td>
</tr>
<tr>
<td>a3</td>
<td>6.0</td>
<td>3</td>
<td>810</td>
<td>34%</td>
<td>2740</td>
<td>a3</td>
<td>2520</td>
</tr>
<tr>
<td>a4</td>
<td>405.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>a4</td>
<td>2740</td>
</tr>
<tr>
<td>a5</td>
<td>405.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>a5</td>
<td>5260</td>
</tr>
</tbody>
</table>

f^3 = (2/3) f^2 + (1/3) f^{SNL}

<table>
<thead>
<tr>
<th>k=3</th>
<th>c(f^{k-1})</th>
<th>Path #</th>
<th>g</th>
<th>P</th>
<th>h</th>
<th>f^{SNL}</th>
<th>f^k</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>541.0</td>
<td>1</td>
<td>1127</td>
<td>0%</td>
<td>28</td>
<td>a1</td>
<td>4014</td>
</tr>
<tr>
<td>a2</td>
<td>288.9</td>
<td>2</td>
<td>830</td>
<td>50%</td>
<td>3986</td>
<td>a2</td>
<td>3986</td>
</tr>
<tr>
<td>a3</td>
<td>44.7</td>
<td>3</td>
<td>830</td>
<td>50%</td>
<td>3986</td>
<td>a3</td>
<td>28</td>
</tr>
<tr>
<td>a4</td>
<td>288.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>a4</td>
<td>3986</td>
</tr>
<tr>
<td>a5</td>
<td>541.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>a5</td>
<td>4014</td>
</tr>
</tbody>
</table>

iteration k

<table>
<thead>
<tr>
<th>k</th>
<th>c(f^{k-1})</th>
<th>g</th>
<th>P</th>
<th>h</th>
<th>f^{SNL}</th>
<th>f^k</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>459.5</td>
<td>h1</td>
<td>931.0</td>
<td>7%</td>
<td>527</td>
<td>a1</td>
</tr>
<tr>
<td>a2</td>
<td>354.0</td>
<td>h2</td>
<td>813.5</td>
<td>47%</td>
<td>3736</td>
<td>a2</td>
</tr>
<tr>
<td>a3</td>
<td>12.0</td>
<td>h3</td>
<td>813.5</td>
<td>47%</td>
<td>3736</td>
<td>a3</td>
</tr>
<tr>
<td>a4</td>
<td>354.0</td>
<td>h4</td>
<td>813.5</td>
<td>47%</td>
<td>3736</td>
<td>a4</td>
</tr>
<tr>
<td>a5</td>
<td>459.5</td>
<td>h5</td>
<td>813.5</td>
<td>47%</td>
<td>3736</td>
<td>a5</td>
</tr>
</tbody>
</table>

c(f^{k-1}) = c(f^k)

f^{k-1} = f^k
ASSIGNMENT MODELS
Congested network – Equilibrium models

MSA-FA ALGORITHM
Example (4/4)
Distance from equilibrium vector $f^*$ vs. convergence test $|f^k_{SNL} - f^{k-1}|/|f^{k-1}|$

* $k$-reset every 5 iterations

ASSIGNMENT MODELS
Classification factors

- SUPPLY MODELS
  ✓ WITHIN-DAY STATIC
  ✓ WITHIN-DAY DYNAMIC

- DEMAND MODELS
  ✓ WITHIN-DAY STATIC
  ✓ WITHIN-DAY DYNAMIC

- ASSIGNMENT MODELS
  ✓ Not Congested Systems
  ✓ Congested Systems
    ➢ Equilibrium
    ➢ Dynamic Processes
ASSIGNMENT MODELS
Congested network – Dynamic process models

**SUPPLY MODEL**
- Path Flows
- Network Flow Propagation Model
- Link Flows
- Link Performance Model

**DEMAND MODEL**
- Cost Updating Model
- O-D Expected Path Cost
- Path / Departure Time Choice Model
- Path Flows
- Actual Path costs

**Path Performance Model**

- **Supply**
  - $c_{act}^{t-1} = c(f^{t-1})$
  - $g_{act}^{t-1} = \Delta f c_{act}^{t-1}$

- **Demand**
  - $h^t = \alpha P(g_{pre}^{t-1}) d + (1-\alpha) h^{t-1}$
  - $f^t = \Delta h^t$
  - $c_{act}^t = c(f^t)$
  - $g^t_{act} = \Delta f c_{act}^t$

**WITHIN-DAY STATIC**
- $g_{pre}^t = \beta g_{act}^{t-1} + (1-\beta) g_{pre}^{t-1}$

**WITHIN-DAY DYNAMIC**
- $h_{j}^{t} = \alpha P(g_{pre}^{t-1}) d + (1-\alpha) h_{j}^{t-1}$
- $f_{j}^{t} = \sum_{i=g} \Delta l_{j} (t_{i}, \ldots, t_{j}) P(j) d$
- $t_{act}^{t} = t(f^{t})$
- $TT_{act}^{t} = \Gamma (t^{t})$

Department of Transportation Engineering
University of Naples “Federico II”
ASSIGNMENT MODELS

Acronyms for dynamic process assignment models in the literature

DPA  Dynamic Process Assignment

DPDA  Dynamic Process Dynamic Assignment

ASSIGNMENT MODELS

Congested network – Dynamic process models

(pre-trip information system)
ASSIGNMENT MODELS
Congested network – Dynamic process models

SUPPLY MODELS
DEMAND MODELS

PREDICTIVE INFORMATION
CONSISTENCY LOOPS

ASSIGNMENT MODELS
Congested network – Dynamic process models
ASSIGNMENT MODELS

the concept of “over-reaction”

Providing predictive information not consistent with drivers’ behavioral response (no consistency loop) may cause worsening of network performances

Example (i)

Usual network conditions

<table>
<thead>
<tr>
<th></th>
<th>free-flow travel time</th>
<th>free flow path choice probability</th>
<th>Equilibrium path choice probability</th>
<th>flow at equilibrium</th>
<th>Travel time at equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>path 1</td>
<td>10</td>
<td>80%</td>
<td>70%</td>
<td>700</td>
<td>12</td>
</tr>
<tr>
<td>path 2</td>
<td>15</td>
<td>20%</td>
<td>30%</td>
<td>300</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1000</td>
</tr>
</tbody>
</table>

d_{cd}=1000 veh/h

Example (ii)

Accident occurring on path 1:
Travel time info on path 1: 12 ➔ 20 min

<table>
<thead>
<tr>
<th></th>
<th>Travel time at equilibrium</th>
<th>not informed drivers</th>
<th>informed drivers</th>
<th>total flow</th>
<th>travel time with accident</th>
</tr>
</thead>
<tbody>
<tr>
<td>path 1</td>
<td>12</td>
<td>665</td>
<td>665</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>path 2</td>
<td>15</td>
<td>285</td>
<td>50</td>
<td>335</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>950</td>
<td>50</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Market penetration (i.e. % of informed users) 5%

Travel time reduction for Informed Drivers: 4 minutes
ASSIGNMENT MODELS
the concept of “over-reaction”

Providing predictive information not consistent with drivers’ behavioral response (no consistency loop) may cause worsening of network performances

Example (iii) $d_{od}=1000 \text{ veh/h}$

Accident occurring on path 1:
Travel time info on path 1: 12 $\rightarrow$ 20 min

<table>
<thead>
<tr>
<th></th>
<th>Travel time at equilibrium</th>
<th>not informed</th>
<th>informed</th>
<th>total flow</th>
<th>travel time with accident</th>
</tr>
</thead>
<tbody>
<tr>
<td>path 1</td>
<td>12</td>
<td>525</td>
<td>525</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td>path 2</td>
<td>15</td>
<td>225</td>
<td>250</td>
<td>17</td>
<td></td>
</tr>
</tbody>
</table>

Market penetration (i.e. % of informed users) 25%

Travel time reduction: for Informed Drivers 2 minutes

Example (iv) $d_{od}=1000 \text{ veh/h}$

Accident occurring on path 1:
Travel time info on path 1: 12 $\rightarrow$ 20 min

<table>
<thead>
<tr>
<th></th>
<th>Travel time at equilibrium</th>
<th>not informed</th>
<th>informed</th>
<th>total flow</th>
<th>travel time with accident</th>
</tr>
</thead>
<tbody>
<tr>
<td>path 1</td>
<td>12</td>
<td>350</td>
<td>350</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>path 2</td>
<td>15</td>
<td>150</td>
<td>500</td>
<td>25</td>
<td></td>
</tr>
</tbody>
</table>

Market penetration (i.e. % of informed users) 50%

Travel time increase for Informed Drivers: 7 minutes
ASSIGNMENT MODELS

Classification factors

- **SUPPLY MODELS**
  - WITHIN-DAY STATIC
  - WITHIN-DAY DYNAMIC

- **DEMAND MODELS**
  - WITHIN-DAY STATIC
  - WITHIN-DAY DYNAMIC

- **ASSIGNMENT MODELS**
  - Not Congested Systems
  - Congested Systems
    - Equilibrium
    - Dynamic Processes
      Deterministic vs. Stochastic

ASSIGNMENT MODELS

Congested network – Dynamic process models

**DETERMINISTIC PROCESSES**

The state in each period deterministically depends on previous states
(no information - within-day static example)

\[ g_{\text{pre}}^t = g_{\text{exp}}^t \]
\[ h^t = P(g_{\text{pre}}^t) h^{t-1} \]
\[ g_{\text{act}}^t = \Delta^T c(\Delta h^t) \]
\[ f^t = \Delta h^t \]

\[ t \]

Department of Transportation Engineering
University of Naples “Federico II”
ASSIGNMENT MODELS
Congested network – Dynamic process models

DETERMINISTIC PROCESSES
Example
Demand  \(d(1,4)= 1500 \text{ veic/h}\)
Free flow link cost

\[
g_{\text{act}}^t = 0.3g_{\text{act}}^{t-1} + 0.7g_{\text{pre}}^{t-1} \quad (\text{i.e. } \beta=0.3)
\]
\[
h^t = P(g^t) h^{t-1} \quad (\text{i.e. } \alpha=1)
\]
\[
P(g^t) = \frac{\exp[-(1/60) \cdot g^t]}{\sum \exp[-(1/60) \cdot g^t]}
\]
\[
\sum = \Delta h^t
\]
\[
f^t = \Delta h^t
\]

Assignment Models
Congested network – Dynamic process models

DETERMINISTIC PROCESSES
Example (I)

<table>
<thead>
<tr>
<th>(t=0)</th>
<th>(c_0)</th>
<th>Path #</th>
<th>(g_{\text{act}}^0)</th>
<th>(g_{\text{pre}}^0)</th>
<th>(P)</th>
<th>(h^1)</th>
<th>(f^1)</th>
<th>(c(f^1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a1)</td>
<td>5.0</td>
<td>1</td>
<td>15</td>
<td>33%</td>
<td>500</td>
<td>(a1)</td>
<td>1000</td>
<td>30.0</td>
</tr>
<tr>
<td>(a2)</td>
<td>10.0</td>
<td>2</td>
<td>15</td>
<td>33%</td>
<td>500</td>
<td>(a2)</td>
<td>500</td>
<td>16.3</td>
</tr>
<tr>
<td>(a3)</td>
<td>5.0</td>
<td>3</td>
<td>15</td>
<td>33%</td>
<td>500</td>
<td>(a3)</td>
<td>500</td>
<td>11.3</td>
</tr>
<tr>
<td>(a4)</td>
<td>10.0</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td>(a4)</td>
<td>500</td>
<td>16.3</td>
</tr>
<tr>
<td>(a5)</td>
<td>5.0</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td>(a5)</td>
<td>1000</td>
<td>30.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(t=1)</th>
<th>(c^{t+1})</th>
<th>Path #</th>
<th>(g_{\text{act}}^{t+1})</th>
<th>(g_{\text{pre}}^{t+1})</th>
<th>(P)</th>
<th>(h^1)</th>
<th>(f^1)</th>
<th>(c(f^1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a1)</td>
<td>30.0</td>
<td>1</td>
<td>71.3</td>
<td>31.2</td>
<td>31%</td>
<td>459</td>
<td>(a1)</td>
<td>980</td>
</tr>
<tr>
<td>(a2)</td>
<td>16.3</td>
<td>2</td>
<td>46.3</td>
<td>24.4</td>
<td>35%</td>
<td>520</td>
<td>(a2)</td>
<td>520</td>
</tr>
<tr>
<td>(a3)</td>
<td>73.3</td>
<td>3</td>
<td>46.3</td>
<td>24.4</td>
<td>35%</td>
<td>520</td>
<td>(a3)</td>
<td>459</td>
</tr>
<tr>
<td>(a4)</td>
<td>16.3</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td>(a4)</td>
<td>520</td>
<td>16.8</td>
</tr>
<tr>
<td>(a5)</td>
<td>30.0</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td>(a5)</td>
<td>980</td>
<td>29.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(t=2)</th>
<th>(c^{t+1})</th>
<th>Path #</th>
<th>(g_{\text{act}}^{t+1})</th>
<th>(g_{\text{pre}}^{t+1})</th>
<th>(P)</th>
<th>(h^1)</th>
<th>(f^1)</th>
<th>(c(f^1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a1)</td>
<td>29.0</td>
<td>1</td>
<td>68.3</td>
<td>42.2</td>
<td>28%</td>
<td>436</td>
<td>(a1)</td>
<td>968</td>
</tr>
<tr>
<td>(a2)</td>
<td>16.8</td>
<td>2</td>
<td>45.8</td>
<td>30.8</td>
<td>35%</td>
<td>532</td>
<td>(a2)</td>
<td>532</td>
</tr>
<tr>
<td>(a3)</td>
<td>16.8</td>
<td>3</td>
<td>45.8</td>
<td>30.8</td>
<td>35%</td>
<td>532</td>
<td>(a3)</td>
<td>456</td>
</tr>
<tr>
<td>(a4)</td>
<td>16.8</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td>(a4)</td>
<td>532</td>
<td>17.1</td>
</tr>
<tr>
<td>(a5)</td>
<td>29.0</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td>(a5)</td>
<td>968</td>
<td>28.4</td>
</tr>
</tbody>
</table>

Department of Transportation Engineering
University of Naples “Federico II”
ASSIGNMENT MODELS

Congested network – Dynamic process models

DETERMINISTIC PROCESSES

Example (I)

<table>
<thead>
<tr>
<th>$t=25$</th>
<th>$c^{t-1}$</th>
<th>Path #</th>
<th>$g^t_{act}$</th>
<th>$g^t_{pre}$</th>
<th>$P$</th>
<th>$h^t$</th>
<th>$f^t$</th>
<th>$c(f^t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>27.6</td>
<td>1</td>
<td>64.2</td>
<td>64.1</td>
<td>27%</td>
<td>400</td>
<td>a1</td>
<td>950</td>
</tr>
<tr>
<td>a2</td>
<td>17.6</td>
<td>2</td>
<td>45.1</td>
<td>45.1</td>
<td>37%</td>
<td>550</td>
<td>a2</td>
<td>550</td>
</tr>
<tr>
<td>a3</td>
<td>9.6</td>
<td>3</td>
<td>45.1</td>
<td>45.1</td>
<td>37%</td>
<td>550</td>
<td>a3</td>
<td>400</td>
</tr>
<tr>
<td>a4</td>
<td>17.6</td>
<td>4</td>
<td>550</td>
<td>18</td>
<td></td>
<td></td>
<td>a4</td>
<td>550</td>
</tr>
<tr>
<td>a5</td>
<td>27.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>a5</td>
<td>950</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$t=26$</th>
<th>$c^{t-1}$</th>
<th>Path #</th>
<th>$g^t_{act}$</th>
<th>$g^t_{pre}$</th>
<th>$P$</th>
<th>$h^t$</th>
<th>$f^t$</th>
<th>$c(f^t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>27.6</td>
<td>1</td>
<td>64.2</td>
<td>64.2</td>
<td>27%</td>
<td>400</td>
<td>a1</td>
<td>950</td>
</tr>
<tr>
<td>a2</td>
<td>17.6</td>
<td>2</td>
<td>45.1</td>
<td>45.1</td>
<td>37%</td>
<td>550</td>
<td>a2</td>
<td>550</td>
</tr>
<tr>
<td>a3</td>
<td>9.6</td>
<td>3</td>
<td>45.1</td>
<td>45.1</td>
<td>37%</td>
<td>550</td>
<td>a3</td>
<td>400</td>
</tr>
<tr>
<td>a4</td>
<td>17.6</td>
<td>4</td>
<td>550</td>
<td>18</td>
<td></td>
<td></td>
<td>a4</td>
<td>550</td>
</tr>
<tr>
<td>a5</td>
<td>27.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>a5</td>
<td>950</td>
</tr>
</tbody>
</table>

Assessment of path cost variation over time.
**ASSIGNMENT MODELS**

**Congested network – Dynamic process models**

**DETERMINISTIC PROCESSES**

Example (II): Actual vs Pre-trip Path cost day-to-day pattern

Example parameters (→ cfr. example I)

d(1,4)= 8000 veic/h ; β=0.77

<table>
<thead>
<tr>
<th>t=0</th>
<th>Path #</th>
<th>g0</th>
<th>gatr</th>
<th>P</th>
<th>h</th>
<th>f</th>
<th>c(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>5.0</td>
<td>1</td>
<td>15</td>
<td>15</td>
<td>33</td>
<td>2667</td>
<td>5333</td>
</tr>
<tr>
<td>a2</td>
<td>10.0</td>
<td>2</td>
<td>15</td>
<td>15</td>
<td>33</td>
<td>2667</td>
<td>716.1</td>
</tr>
<tr>
<td>a3</td>
<td>5.0</td>
<td>3</td>
<td>15</td>
<td>15</td>
<td>33</td>
<td>2667</td>
<td>448.2</td>
</tr>
<tr>
<td>a4</td>
<td>10.0</td>
<td>4</td>
<td>15</td>
<td>15</td>
<td>33</td>
<td>2667</td>
<td>448.2</td>
</tr>
<tr>
<td>a5</td>
<td>5.0</td>
<td>5</td>
<td>15</td>
<td>15</td>
<td>33</td>
<td>2667</td>
<td>448.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>t=1</th>
<th>Path #</th>
<th>g0</th>
<th>gatr</th>
<th>P</th>
<th>h</th>
<th>f</th>
<th>c(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>10.1</td>
<td>1</td>
<td>1815.0</td>
<td>1247.0</td>
<td>30</td>
<td>0</td>
<td>4000</td>
</tr>
<tr>
<td>a2</td>
<td>82.3</td>
<td>2</td>
<td>903.9</td>
<td>699.4</td>
<td>50</td>
<td>4000</td>
<td>4900</td>
</tr>
<tr>
<td>a3</td>
<td>5.1</td>
<td>3</td>
<td>903.9</td>
<td>699.4</td>
<td>50</td>
<td>4000</td>
<td>4900</td>
</tr>
<tr>
<td>a4</td>
<td>10.0</td>
<td>4</td>
<td>903.9</td>
<td>699.4</td>
<td>50</td>
<td>4000</td>
<td>4900</td>
</tr>
<tr>
<td>a5</td>
<td>5.1</td>
<td>5</td>
<td>903.9</td>
<td>699.4</td>
<td>50</td>
<td>4000</td>
<td>4900</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>t=2</th>
<th>Path #</th>
<th>g0</th>
<th>gatr</th>
<th>P</th>
<th>h</th>
<th>f</th>
<th>c(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>805.0</td>
<td>1</td>
<td>815.0</td>
<td>788.4</td>
<td>32</td>
<td>402</td>
<td>4231</td>
</tr>
<tr>
<td>a2</td>
<td>100.0</td>
<td>2</td>
<td>815.0</td>
<td>788.4</td>
<td>32</td>
<td>402</td>
<td>4231</td>
</tr>
<tr>
<td>a3</td>
<td>5.1</td>
<td>3</td>
<td>815.0</td>
<td>788.4</td>
<td>32</td>
<td>402</td>
<td>4231</td>
</tr>
<tr>
<td>a4</td>
<td>10.0</td>
<td>4</td>
<td>815.0</td>
<td>788.4</td>
<td>32</td>
<td>402</td>
<td>4231</td>
</tr>
<tr>
<td>a5</td>
<td>5.1</td>
<td>5</td>
<td>815.0</td>
<td>788.4</td>
<td>32</td>
<td>402</td>
<td>4231</td>
</tr>
</tbody>
</table>

**ASSIGNMENT MODELS**

**Congested network – Dynamic process models**

**DETERMINISTIC PROCESSES**

Example (II)
ASSIGNMENT MODELS

Congested network – Dynamic process models

DETERMINISTIC PROCESSES

Example (II): Actual vs Pre-trip Path cost day-to-day pattern

Path h1

Path h2, Path h3

DETERMINISTIC PROCESSES

Attractors

A minimal subset of the state space:

• dimension less than the dimension of the state space
• the system may not evolve outside it, from a state inside it
• there is a proper superset (the basin) from which the system evolves toward the attractor

Department of Transportation Engineering
University of Naples “Federico II”
ASSIGNMENT MODELS
Congested network – Dynamic process models

DETERMINISTIC PROCESSES

Stability conditions
The Stability of the attractor decreases when:

* demand flows increase 
* link capacities decrease 

Congestion levels increase 

* the variance of the random residuals decreases \( \rightarrow \) more info 
* parameters \( \alpha \) and \( \beta \) increase \( \rightarrow \) more reactivity

EXAMPLE

Stability regions (i.e. ellipses) of a fixed point state for \( \alpha = \beta \).

ASSIGNMENT MODELS
Congested network – Dynamic process models

DETERMINISTIC PROCESSES

Fixed-point attractors
(no information - within-day static example)

\[ g_{pre}^t = g_{exp}^t \]

RECURSIVE EQUATIONS

\[ g_{pre}^t = g(g_{act}^{t-1} = \Delta^T c(\Delta h^{t-1}), g_{pre}^{t-1}) \]

\[ h^t = P(g_{pre}^t) h^{t-1} \]

FIXED-POINT CONDITIONS

\[ g_{pre}^t = g_{pre}^{t-1} = g_{pre}^* \]

\[ h^t = h^{t-1} = h^* \]

FIXED POINT ATTRACTION

\[ g_{pre}^* = g(g_{act}^* = \Delta^T c(\Delta h^*), g_{pre}^*) \]

\[ h^* = P(g_{pre}^*) h^* \]
ASSIGNMENT MODELS
Congested network – Dynamic process models

DETERMINISTIC PROCESSES

**Fixed-point attractors** \(\rightarrow\) EQUIVALENCE WITH EQUILIBRIUM IF:

- **HOMOGENEOUS COST UPDATING MODELS**
  if the actual path cost at day \(t-1\) is different from what travellers expected:
  \[ g_{act}^{t-1} \neq g_{pre}^{t-1} \]
  such difference implies a different expected path cost at day \(t\)
  \[ g_{pre}^{t} \neq g_{pre}^{t-1} \]

- **EXPONENTIAL SMOOTHING CHOICE UPDATING MODELS**
  \[ g_{act}^{*} = \Delta^{T} c(d \Delta h^{*}) \]
  \[ h^{*} = P(g_{act}^{*}) \]
  \[ f^{*} = \Delta P(\Delta^{T} c(f^{*})) \]

ASSIGNMENT MODELS
Congested network – Dynamic process models

STOCHASTIC PROCESSES
State in each period is a random variable with distribution depending on previous states
(no information - within-day static example)

\[ g_{pre}^{t} = g_{exp}^{t} \]
\[ g_{act}^{t-1} \leftarrow G_{act}^{t-1} \]
with \( E[G_{act}^{t-1}] = \Delta^{T} c(\Delta h^{t-1}) \)
\[ g_{pre}^{t} \leftarrow G_{pre}^{t} \]
with \( E[G_{pre}^{t}] = g(g_{act}^{t-1}, g_{pre}^{t-1}) \)
\[ h^{t} \leftarrow \Delta h^{t} \]
with \( E[H^{t}] = P(g_{pre}^{t}) \)

ASSIGNMENT MODELS

Congested network – Dynamic process models

EQUILIBRIUM VS. DYNAMIC PROCESSES

EQUILIBRIUM MODELS

**ADVANTAGES**
- no explicit modeling of users' cost and choice updating processes
- use of well founded models and algorithms (within-day static case)

**DRAWBACKS**
- uncertain relevance
- stability analysis not meaningful
- no simulation of transients and non recurrent conditions
- no system state statistical description

---

DYNAMIC MODELS

**THEORETICAL ADVANTAGES**
- identification of attractors
- stability analysis

**APPLICATIVE ADVANTAGES**
- simulation of transients and non recurrent conditions
- system state statistical description

**DRAWBACKS**
- require explicit modeling of users' cost and choice updating processes (memory, habit, etc.)
- computational

Dynamic control strategies reacting to perturbations in demand and/or supply can be effectively simulated only through dynamic process models