Estimation of Origin to Destination Flows from Counts

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OUTLINE

• PRELIMINARY CONSIDERATIONS

• PART I: STATIC O-D ESTIMATION

• PART II: DYNAMIC O-D ESTIMATION

• PART III: QUASI-DYNAMIC OD ESTIMATION
• PRELIMINARY CONSIDERATIONS

• PART I: STATIC O-D ESTIMATION

• PART II: DYNAMIC O-D ESTIMATION

• PART III: QUASI-DYNAMIC OD ESTIMATION
PRELIMINARY CONSIDERATIONS

• O-D trip matrices are a fundamental input for most problems related to the management and planning of transportation systems.

• O-D trip matrices were typically estimated by using sample surveys of various types (expansive, time consuming, not easily repeatable, etc.) and/or travel demand models.

• Recently, considerable work has been devoted to improve the quality of O-D demand estimates by using cheap, easily and automatically collectable traffic counts.
PRELIMINARY CONSIDERATIONS

Network model

Path choice model

O-D demand

ASSIGNMENT

Calculated link flows

Path choice model

O-D estimation

Network model

Measured link flows

O-D demand
**PRELIMINARY CONSIDERATIONS**

**Example**

Counted Flows

\[ f_{1,5} = 15 \quad f_{5,6} = 20 \]

\[ f_{\text{sim}}^{1,5} = 19 \quad f_{\text{sim}}^{5,6} = 24 \]

Initial estimate of the O-D flows:

\[
\begin{array}{ccc}
  & 3 & 4 \\
1 & 12 & 7 \\
2 & 3 & 2 \\
\end{array}
\]

**Distance Measures**

<table>
<thead>
<tr>
<th>(d)</th>
<th>(\sum_{od} (d_{od} - \hat{d}_{od})^2)</th>
<th>(\hat{d}_{od})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d_1)</td>
<td>66</td>
<td>8.702</td>
</tr>
<tr>
<td>(d_2)</td>
<td>26</td>
<td>2.226</td>
</tr>
<tr>
<td>(d_3)</td>
<td>16</td>
<td>2.286</td>
</tr>
</tbody>
</table>
The balance between unknown O-D flows $d_{od}$ and “independent” information about counts $n_i$

\[
\hat{f}_{1,5} = 15 \\
\hat{f}_{5,6} = 20
\]

\[
f_{1,5} = d_{13} + d_{14}
\]

\[
f_{5,6} = d_{13} + d_{14} + d_{23} + d_{24}
\]
## PRELIMINARY CONSIDERATIONS

<table>
<thead>
<tr>
<th>Application</th>
<th>Estimation</th>
<th>Prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>off-line</strong></td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>to design and evaluate traffic</td>
<td></td>
<td></td>
</tr>
<tr>
<td>management schemes and systems</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>on-line</strong></td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>to support operation of real-time</td>
<td></td>
<td></td>
</tr>
<tr>
<td>control strategies (e.g. predictive</td>
<td></td>
<td></td>
</tr>
<tr>
<td>control)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- to update an estimate of OD matrix using observed link flows (counts)
- to predict OD matrices for future time slices combining historical OD matrices, OD matrices estimated for previous time slices, and observed link flows
OUTLINE

• PRELIMINARY CONSIDERATIONS

• PART I: STATIC O-D ESTIMATION

• PART II: DYNAMIC O-D ESTIMATION

• PART III: QUASI-DYNAMIC OD ESTIMATION
Contents

• Notation and terminology
• Relationship between traffic counts and O-D demand flows
• Estimators of O-D trip matrices
• Solution methods
STATIC O-D ESTIMATION

Notation and terminology

Static Supply Model

- **GRAPH**

Ordered pair of sets $G(N,L)$ representing network connections

$N$: *set of nodes* $\equiv \{1, 2, 3, 4\}$

$L$: *set of links* $\equiv \{a_1, a_2, a_3, a_4, a_5\}$

Origin centroid: 1
Destination centroid: 4

- **Path $k$**:

Sequence of links representing “typical” trips allowed by the supply system modeled

Path set $K \equiv \{1, 2, 3\}$
### Static O-D Estimation

**Notation and terminology**

**Static Supply Model**

- **Link-Path Incidence Matrix**
  
  \( \delta_{ak} = 1 \) if link \( a \) belongs to path \( k \),
  
  \( \delta_{ak} = 0 \) otherwise

  \( \Delta = [\delta_{ak}]_{ak} = \text{link-path incidence matrix} \)

#### Table

<table>
<thead>
<tr>
<th>O-D</th>
<th>1-4</th>
<th>2-4</th>
<th>3-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>paths</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>links</td>
<td>a_1</td>
<td>a_2</td>
<td>a_3</td>
</tr>
<tr>
<td>a_1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>a_2</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>a_3</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>a_4</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>a_5</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

**Example:**

1\(^{st}\) row: link \( a_1 \) belongs to paths 1 and 3 \( (\delta_{a_11} = \delta_{a_13} = 1) \)

1\(^{st}\) column: path 1 crosses links \( a_1, a_3 \) and \( a_5 \) \( (\delta_{a_11} = \delta_{a_31} = \delta_{a_51} = 1) \)
Simulation period = simulation life = 1 hour

- **PATH FLOWS**
  \( h_k \): n. of trips using path \( k \), connecting O-D pair \( (o,d) \), in a fixed time period \( (k \in I_{od}) \).

\[
\begin{align*}
\Rightarrow & \quad h_1 = 800 \text{ veic/h} \\
\Rightarrow & \quad h_2 = 1200 \text{ veic/h} \\
\Rightarrow & \quad h_3 = 400 \text{ veic/h} \\
\end{align*}
\]

\[2.400 \text{ veic/h}\]

Path flows are propagated simultaneously on each link belonging to the path
STATIC O-D ESTIMATION

Notation and terminology

Static Supply Model

Network flow propagation

- **LINK FLOWS**: *n. of trips using link a in a fixed time period.*
Static Supply Model

- LINK COST

\[ c_a \Rightarrow \text{generalized transportation cost of link } a, \text{ i.e. a function of the } n'\text{s disutility (performance) attributes } r_{n,a} \text{ perceived by travelers in making their choices} \]

\[ c_a = c(f) = \sum_n \beta_n r_{n,a}(f) \]
STATIC O-D ESTIMATION

Notation and terminology

Static Supply Model

• LINK COST

Attributes making up generalized transportation cost of link $a$:

- Average Travel Time
  - Running Time
  - Waiting Time
- Out-of-pocket money
  - Tolls
  - Gasoline

Driving quality (e.g. information systems, etc.)

Scenery $f_a/Q_a$ (e.g. aesthetic attributes)
Static Supply Model

PATH COSTS

\( g_k \rightarrow \text{cost of the path } k \)

\[ g_1 = c_{a_1} + c_{a_3} + c_{a_5} \]

\[ g_2 = c_{a_2} + c_{a_5} \]

\[ g_3 = c_{a_1} + c_{a_4} \]

Path costs are not always additive w. r. t. link costs!
STATIC O-D ESTIMATION

Notation and terminology

Static Supply Model

Set of equations relating path costs and other intermediate variables (link flows, travel times, etc.) to path flows

taken from:
STATIC O-D ESTIMATION

Notation and terminology

Static Supply Model

OVERALL SUPPLY MODEL

GRAPH MODEL

\((N,L) ; \Delta\)

NETWORK FLOW PROPAGATION MODEL

\(f = \Delta \cdot h\)

LINK PERFORMANCE MODEL

\(c_a = c(f) = \sum_n \beta_n \cdot r_{n,a}(f)\)

PATH PERFORMANCE MODEL

\(g = \Delta^T \cdot c\)

\(g = \Delta^T \cdot c(\Delta \cdot h)\)
STATIC O-D ESTIMATION

Notation and terminology

Static Supply Model

O-D = (1,4)

Paths:

Path performance model

\[
g = \Delta^T
\]

Network flow propagation model

\[
f = \Delta
\]

\[
\begin{bmatrix}
g_1 \\
g_2 \\
g_3
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 & 0
\end{bmatrix} \begin{bmatrix}
ca_1 \\
ca_2 \\
ca_3 \\
ca_4 \\
ca_5
\end{bmatrix}
\]

\[
\begin{bmatrix}
fa_1 \\
fa_2 \\
fa_3 \\
fa_4 \\
fa_5
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 1 \\
1 & 1 & 0
\end{bmatrix} \begin{bmatrix}
h_1 \\
h_2 \\
h_3
\end{bmatrix}
\]
Path choice models

DETERMINISTIC
Users choose only minimum cost alternatives

PROBABILISTIC
Users may choose any available alternative with a probability depending on costs
STATIC O-D ESTIMATION

Notation and terminology

Example of Path probability matrix $P$

$$g^T = [g1, g2, g3, g4, g5, g6]$$

$$P_{od,k} = \frac{\exp(-g_{od,k} / \theta)}{\sum_{j \in K_{od}} \exp(-g_{od,j} / \theta)};$$

$$\theta = 2$$

$$p_{1/14} + p_{2/14} + p_{3/14} = 1$$

<table>
<thead>
<tr>
<th>Path/OD</th>
<th>1-4</th>
<th>2-4</th>
<th>3-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$P_{1/14}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>$P_{2/14}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>$P_{3/14}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>$P_{4/2-4}$</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>$P_{5/2-4}$</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
<td>$P_{6/3-4}$</td>
</tr>
</tbody>
</table>
Notation and terminology

• **O-D Trip vector** \( d \equiv \{ d_{od} \} \quad o, d \in N_x \)
  
  \( d_{od} \): average number of trips going from origin \( o \) to destination \( d \) with a given mode within a given time period.

• **Path flow vector** \( h \equiv \{ h_k \} \)
  
  \( h_k \): number of trips using path \( k \), connecting O-D pair \((o,d)\), in a fixed time period (\( k \in I_{od} \)).

• **Path choice fraction matrix** \( P \equiv \{ p_{k,od} \} \)
  
  \( p_{k,od} \): \( p(k/od) \) fraction of trips using path \( k \) (connecting O-D pair \((o,d)\)) in a fixed time period.

• **Link-path incidence matrix** \( \Delta \equiv \{ \delta_{lk} \} \)
  
  \( \delta_{lk} \): 1 if link \( l \) belongs to path \( k \); = 0 otherwise.

• **Assignment matrix** \( M = \Delta P \equiv \{ m_{l,od} \} \)
  
  \( m_{l,od} \): fraction of O-D flow \( d_{od} \) contributing to \( f_l \).
STATIC O-D ESTIMATION

Relationship between traffic counts and O-D demand flows

• EXAMPLE OF ASSIGNMENT MATRIX

\[ d = \begin{bmatrix} d_{13} \\ d_{23} \end{bmatrix} = \begin{bmatrix} 100 \\ 80 \end{bmatrix} \]

<table>
<thead>
<tr>
<th>O-D pair</th>
<th>Path k</th>
<th>( p_{k,od} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-3</td>
<td>1) 1 4 6 7 9 3</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>2) 1 4 5 7 9 3</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>3) 1 4 5 7 9 8 3</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>4) 1 4 6 7 9 8 3</td>
<td>0.20</td>
</tr>
<tr>
<td>2-3</td>
<td>5) 2 5 7 9 3</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>6) 2 5 7 9 8 3</td>
<td>0.30</td>
</tr>
</tbody>
</table>

\( N = 2 \) (link 9-3 e link 8-3)

\[ \Delta \cdot P = M \]

\[
\begin{bmatrix}
1-3 & 2-3 \\
9-3 & 1 & 1 & 1 \\
8-3 & 1 & 1 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0.3 \\
2 & 0.3 \\
3 & 0.2 \\
4 & 0.2 \\
5 & 0.7 \\
6 & 0.3
\end{bmatrix}
\]

\[
\begin{bmatrix}
1-3 & 2-3 \\
9-3 & 0.6 & 0.7 \\
8-3 & 0.4 & 0.3
\end{bmatrix}
\]
STATIC O-D ESTIMATION

Relationship between traffic counts and O-D demand flows

\[ f_l = \sum_k \delta_{lk} h_k \]

\[ f_l = \sum_k \delta_{lk} h_k = \sum_k \delta_{lk} \sum_{od} p_{k,od} d_{od} \]

\[ f_l = \sum_{od} d_{od} \sum_k \delta_{lk} p_{k,od} = \sum_{od} m_{l,od} d_{od} \]

\[ f_l = m_l^T d \]

• ASSIGNMENT MAP

\[ f = \Delta h = \Delta Pd = M d \]

Example
Relationship between traffic counts and O-D demand flows

\[ f = M d \]  

\[ d = \begin{bmatrix} d_{13} \\ d_{23} \end{bmatrix} = \begin{bmatrix} 100 \\ 80 \end{bmatrix} \]  

\[ f = M d \]

\[
\begin{bmatrix}
9-3 & 116 \\
8-3 & 64
\end{bmatrix}
= \begin{bmatrix}
1-3 & 2-3 \\
9-3 & 0.6 & 0.7 \\
8-3 & 0.4 & 0.3 \\
1-3 & 100 \\
2-3 & 80
\end{bmatrix}
\]
The balance between unknown O-D flows $d_{od}$ and “independent” information about counts $n_l$ for real networks

$$n_l = 1,000 \div 10,000$$

$$n_{od} = \left(200 \times 200\right) \div \left(1000 \times 1000\right)$$
STATIC O-D ESTIMATION

Relationship between traffic counts and O-D demand flows

- $\hat{M} = \text{estimate of } M \text{ from the assignment model}$
- $g = \text{overall path cost vector}$

\[
\hat{p}_{k,od} = p[k/od](g)
\]

\[
g = \Delta^T c
\]

\[
\hat{M}(c) = \Delta \hat{P}(c)
\]

\[
v = \hat{M}d = v(d)
\]

\[
\hat{M} = M + E
\]

\[
f = (\hat{M} - E)d = \hat{M}d - Ed
\]

\[
Ed \Rightarrow \varepsilon_{SIM}
\]

\[
f = \hat{M}d + \varepsilon_{SIM}
\]
STATIC O-D ESTIMATION

Relationship between traffic counts and O-D demand flows

- Observed link flow vector (traffic counts) \( \hat{f} = \{ \hat{f}_i \} \):

\[
\hat{f} = f + \varepsilon^{OBS}
\]

\[
\hat{f} = \hat{M}d + \varepsilon^{SIM} + \varepsilon^{OBS} = v(d) + \varepsilon
\]

\( \varepsilon \): vector of random error terms with \( E(\varepsilon) = 0 \)
- measurement errors
- temporal fluctuation of demand
- assignment model (path choice and network) errors
- etc.
STATIC O-D ESTIMATION

Estimators of O-D trip demand

• PROBLEM STATEMENT
  Estimate the O/D trip demand $d$ by “efficiently” combining traffic counts with all other available information

• ESTIMATION PROBLEMS
  – Experimental information (sample surveys) + traffic counts
    "CLASSIC" INFERENCE

  – Non-experimental information ("a priori" information) + traffic counts
    BAYESIAN INFERENCE
STATIC O-D ESTIMATION

Estimators of O-D trip demand

GENERAL ESTIMATION PROBLEM

Several models have been proposed to estimate O-D flows by combining counts with other information sources.

General Formulation

\[ d^* = \arg\min_{x \in S} [z_1(x, \hat{d}) + z_2(v(x), \hat{f})] \]

where:

- \( z_1(x, \hat{d}) \) is a measure of the “distance” between the unknown demand \( x \) and the a priori demand \( d \);

- \( z_2(v(x), \hat{f}) \) is a measure of the “distance” between the link flows resulting from the unknown demand \( x \) and the observed link flows \( f \).
# STATIC O-D ESTIMATION

## Estimators of O-D trip demand

### POSSIBLE FUNCTIONAL FORMS for $z_1(.)$ and $z_2(.)$

<table>
<thead>
<tr>
<th>Distance from the initial estimate $z_1(x, \hat{d})$</th>
<th>Distance from flows counts $z_2(v(x), \hat{f})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generalized least squares (GLS) $(\hat{d} - x)^T Z^{-1}(d - x)$ or $\sum_{od} (x_{od} - \hat{d}<em>{od})^2 / \text{Var}[\eta</em>{od}]$</td>
<td>Generalized least squares (GLS) $(\hat{f} - v(x))^T W^{-1}(\hat{f} - v(x))$ or $\sum_{i \in M} (\hat{f}_i - v_i(x)) / \text{Var}[\epsilon_i]$</td>
</tr>
<tr>
<td>Maximum Likelihood (ML) Poisson $-\sum_{od} (n_{od} \ln(\alpha_{ods} x_{od}) - \alpha_{ods} x_{od})$</td>
<td>Maximum Likelihood (ML) Poisson $\sum_{i \in M} (\hat{f}_i \ln v_i(x) - v_i(x))$</td>
</tr>
<tr>
<td>MVN $(\hat{d} - x)^T Z^{-1}(\hat{d} - x)$ or $\sum_{od} (x_{od} - \hat{d}<em>{od})^2 / \text{Var}[\eta</em>{od}]$</td>
<td>MVN $(\hat{f} - v(x))^T W^{-1}(\hat{f} - v(x))$ or $\sum_{i \in M} (\hat{f}_i - v_i(x)) / \text{Var}[\epsilon_i]$</td>
</tr>
<tr>
<td>Bayes Poisson $\sum_{od} x_{od} \ln(x_{od} / \hat{d}_{od} - 1)$</td>
<td>Bayes Poisson $\sum_{i \in M} (\hat{f}_i \ln v_i(x) - v_i(x))$</td>
</tr>
<tr>
<td>MVN $(\hat{d} - x)^T Z^{-1}(\hat{d} - x)$ or $\sum_{od} (x_{od} - \hat{d}<em>{od})^2 / \text{Var}[\eta</em>{od}]$</td>
<td>MVN $(\hat{f} - v(x))^T W^{-1}(\hat{f} - v(x))$ or $\sum_{i \in M} (\hat{f}_i - v_i(x)) / \text{Var}[\epsilon_i]$</td>
</tr>
<tr>
<td>Multinomial $\sum_{od} x_{od} \ln(x_{od} / \hat{d}_{od})$</td>
<td></td>
</tr>
<tr>
<td>$\sum_{od} x_{od} = \text{const.}$</td>
<td></td>
</tr>
</tbody>
</table>

**Note:** The table provides various functional forms for estimating O-D trip demand. The forms include generalized least squares (GLS), maximum likelihood (ML) for Poisson and MVN distributions, and Bayes estimators for Poisson and MVN distributions. The expressions are structured to show how distance from the initial estimate and distance from flows counts are calculated and minimized under different estimations methods.
GLS ESTIMATORS

Experimental and numerical tests show that Generalized Least Square method is robust.

\[ \hat{d} = x + \eta \quad \text{E}(\eta) = 0 \quad \text{Var}(\eta) = Z \]

\[ \hat{f} = \hat{M}x + \varepsilon \quad \text{E}(\varepsilon) = 0 \quad \text{Var}(\varepsilon) = W \]

\[ d^{\text{GLS}} = \arg\min_{x \in S} \left[ (\hat{d} - x)^T Z^{-1} (\hat{d} - x) + (\hat{f} - \hat{M}x)^T W^{-1} (\hat{f} - \hat{M}x) \right] \]

Simplified Version:

\[ d^{\text{GLS}} = \arg\min_{x \geq 0} \left[ \sum_{od} \frac{(\hat{d}_{od} - x_{od})^2}{\text{var}[\eta_{od}]} + \sum_{l} \frac{(\hat{f}_l - \sum_{od} \hat{m}_{l,od} x_{od})^2}{\text{var}[\varepsilon_l]} \right] \]
STATIC O-D ESTIMATION
Solution methods

- APPLICATION

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Nodes</td>
<td>2.903</td>
</tr>
<tr>
<td>Centroids</td>
<td>167</td>
</tr>
<tr>
<td>Road Links</td>
<td>5.102</td>
</tr>
<tr>
<td>Connector links</td>
<td>646</td>
</tr>
<tr>
<td>O-D pairs</td>
<td>27.889</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Counts (7:30-8:30 a.m.)</th>
<th>82</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hold-out counts</td>
<td>20</td>
</tr>
</tbody>
</table>

\[ \hat{d} \] is an out dated estimate of the O-D table
STATIC O-D ESTIMATION

Solution methods

• APPLICATION
STATIC O-D ESTIMATION

Solution methods

• APPLICATION → RESULTS (A)

Statistics

\[
MSE(f^*, \hat{f}) = \frac{\sum_l (f^*_l - \hat{f}_l)^2}{n_l}
\]

\[
RMSE\% = \left( \frac{MSE(f^*, \hat{f})^{1/2}}{\sum_l \hat{f}_l / n_l} \right)
\]

<table>
<thead>
<tr>
<th></th>
<th>Before updating</th>
<th>After updating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counted flows</td>
<td>Assigned flows</td>
<td>Assigned flows</td>
</tr>
<tr>
<td>0</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>500</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>1000</td>
<td>1500</td>
<td>1500</td>
</tr>
<tr>
<td>1500</td>
<td>2000</td>
<td>2000</td>
</tr>
<tr>
<td>2000</td>
<td>2500</td>
<td>2500</td>
</tr>
<tr>
<td>2500</td>
<td>3000</td>
<td>3000</td>
</tr>
<tr>
<td>3000</td>
<td>3500</td>
<td>3500</td>
</tr>
</tbody>
</table>

MSE before: 1,409,442
MSE after: 110,822
% reduction of MSE: -92%

RMSE% before: 0.52
RMSE% after: 0.21
% red. of RMSE%: -59%
STATIC O-D ESTIMATION

Solution methods

- APPLICATION $\rightarrow$ RESULTS (B)
  - Validation with hold-out counts

### Validation Results

<table>
<thead>
<tr>
<th></th>
<th>Before Updating</th>
<th>After Updating</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSE</td>
<td>1,581,023</td>
<td>68,456</td>
</tr>
<tr>
<td>% reduction of MSE</td>
<td>-96%</td>
<td></td>
</tr>
<tr>
<td>RMSE%</td>
<td>1.10</td>
<td>0.25</td>
</tr>
<tr>
<td>% red. of RMSE%</td>
<td>-77%</td>
<td></td>
</tr>
</tbody>
</table>
## STATIC O-D ESTIMATION

**Solution methods**

**APPLICATION ➔ RESULTS (B)**

Validation with hold-out counts

(2/2)

<table>
<thead>
<tr>
<th>INITIAL O/D</th>
<th>INT</th>
<th>EX</th>
<th>TOT</th>
</tr>
</thead>
<tbody>
<tr>
<td>INT</td>
<td>39,355</td>
<td>31,338</td>
<td>70,693</td>
</tr>
<tr>
<td>EX</td>
<td>43,942</td>
<td>13,454</td>
<td>57,396</td>
</tr>
<tr>
<td>TOT</td>
<td>83,297</td>
<td>44,792</td>
<td>128,089</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>ESTIMATED O/D</th>
<th>INT</th>
<th>EX</th>
<th>TOT</th>
</tr>
</thead>
<tbody>
<tr>
<td>INT</td>
<td>47,971</td>
<td>29,497</td>
<td>77,467</td>
</tr>
<tr>
<td>EX</td>
<td>41,256</td>
<td>8,688</td>
<td>49,944</td>
</tr>
<tr>
<td>TOT</td>
<td>89,226</td>
<td>38,185</td>
<td>127,411</td>
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</table>

<table>
<thead>
<tr>
<th>% VAR</th>
<th>INT</th>
<th>EX</th>
<th>TOT</th>
</tr>
</thead>
<tbody>
<tr>
<td>INT</td>
<td>22%</td>
<td>-6%</td>
<td>10%</td>
</tr>
<tr>
<td>EX</td>
<td>-6%</td>
<td>-35%</td>
<td>-13%</td>
</tr>
<tr>
<td>TOT</td>
<td>7%</td>
<td>-15%</td>
<td>-1%</td>
</tr>
</tbody>
</table>
OUTLINE

• PRELIMINARY CONSIDERATIONS

• PART I: STATIC O-D ESTIMATION

• PART II: DYNAMIC O-D ESTIMATION

• PART III: QUASI-DYNAMIC OD ESTIMATION
DYNAMIC O-D ESTIMATION

Extension of previous results to the case of time-varying (within day dynamic) demand and link flows

• Notation and terminology

• Relationship between within-day Dynamic Traffic Counts and O-D demand flows

• Simultaneous dynamic estimators of O/D trip matrices
# DYNAMIC O-D ESTIMATION

## Notation and terminology

### Dynamic Supply Model

<table>
<thead>
<tr>
<th>Flow Representation</th>
<th>Performance functions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>AGGREGATE</strong> (explicit capacity)</td>
</tr>
<tr>
<td><strong>CONTINUOUS</strong></td>
<td><strong>MACRO - SIMULATION</strong></td>
</tr>
<tr>
<td></td>
<td>discrete space</td>
</tr>
<tr>
<td><strong>DISCRETE</strong></td>
<td><strong>MESO - SIMULATION</strong></td>
</tr>
</tbody>
</table>
DYNAMIC O-D ESTIMATION

Notation and terminology

Dynamic Supply Model

• Observation period
  \( n_j \): number of intervals 1, 2.. \( j \),.. \( n_j \)
  \( T \): duration of each interval

• O-D trip vectors \( d_{[t]} \) \{ \( d_{od}[t] \) \}
  \( d_{od}[t] \): number of trips between O-D pair \( o,d \) leaving the origin during interval \( t \)
Dynamic Supply Model

- **Link flow vectors**
  \[ f_j \equiv \{ f_{l[j]} \} \]
  \[ f_{l[j]} : \text{average flow which can be counted on link } l \text{ during interval } j \]

- **Within-day dynamic link flow definition**
  - counting section
  \[ f_{l[j]} = \frac{n(s_c, j)}{T} \]
  - path flow vector \( h_{[t]} = \{ h_k[t] \} \)
  \( h_k[t] : \text{path flow, average number of trips per time unit on path } k \text{ leaving during period } t \)
Dynamic Supply Model

- **Path flows (time dependent)**
  - Network flow propagation

- **Link flows (time dependent)**
  - Time-varying link flows across sections depend on the times at which path flows (packets) reach that section. Moreover, the link flow in a specific interval depends on the section $s$ of the link:

  $$\sum_{k} f_{a,s}^{k}[j]$$
  
  - $f_{a,s}^{k}[j]$ number of users on path $k$ crossing section $s$ of link $a$ during time interval $j$

  $$f_{a,s}[j] = \sum_{k} f_{a,s}^{k}[j]$$
  
  - total flow crossing section $s$ of link $a$ during interval $j$

- $u_{a}[j] = \sum_{k} u_{a}^{k}[j]$ total inflow on link $a$ during interval $j$
Dynamic Supply Model

• **TRAVEL TIME VARIABLE**
  Link travel time \( t_a(\tau) \)
  
  travel time to cross link a entering link a at time \( \tau \)

• **TRAVEL TIME FUNCTIONS**
  Give the forward travel time \( t_a(j) \) of each packet entering/exting link a at a generic time instant of interval j, as a function of, for instance:

  Link density (load) at the characteristic time instant \( \tau j - 1 \) of the previous interval \( j - 1 \)

• **Path choice fraction matrix** \( P_t = \{p(k/t)\} \)
  
  \( p(k/t) \): fraction of O/D flow \( d_{od[t]} \) following path k, given departing interval t

  \[ h_k[t] = p(k \mid t) \cdot d_{od[t]}[t] \]
Dynamic incidence matrix

The dynamic incidence matrix maps path flows departed in interval \( l \) to inflows entering link \( a \) in interval \( j \)

\[
\Delta[l, j] = [\delta_{ak}(l, j)]_{ak} \quad \text{for} \quad l \leq j
\]

\[
\delta_{ak}[l, j] = \begin{cases} 
1 & \text{if} \quad \tau^u_a[k_l] \in ([j-1]DT, [j]DT) \\
0 & \text{otherwise}
\end{cases}
\]

i.e. the dynamic incidence value \( \delta_{ak}(l, j) \) tells us if packet \( k_l \) has entered link \( a \) during time interval \( j \)

\( \delta_{ak}(l, j) \) is a function of the travel times on the links preceding \( a \) on path \( k \)
DYNAMIC O-D ESTIMATION

Notation and terminology

Set of equations relating path costs to path flows

taken from:
OVERALL SUPPLY MODEL

PATH PERFORMANCE MODEL
\[ T_{l} = \sum_{j} \Delta^T(l, j) \cdot t(j) \quad \Delta(l, j) = \Gamma[t(l),...,t(j)] \]

LINK PERFORMANCE MODEL
\[ t(\tau) = t(k(\tau)) \]

NETWORK FLOW PROPAGATION MODEL
\[ u_o[j] = \sum_{i \leq j} \Delta[l,j] \cdot h[l] \]
\[ f[j] = \sum_{l \leq j} \Delta(l,k) \cdot h[l] \]
DYNAMIC O-D ESTIMATION

Relationship between within-day dynamic traffic counts and O-D demand flows

Estimates of variables can be obtained through within-day dynamic assignment models:

• Estimate of path choice fraction matrix $\hat{P}_t$
estimate of departure time/path choice probabilities (from demand model)

• Estimate of the dynamic incidence matrix $\hat{\Delta}_{[t,j]}$
function of link performances (e.g. average speed) on intervals comprised between $t$ and $j$ (see Dynamic Network Flow Propagation Models)

• Estimate of demand assignment matrix $\hat{M}_{[t,j]}$
fractions of O/D flow $d_{od,t}$ contributing to flow on link $l$ during period $j$

$$\hat{M}_{[t,j]} = \hat{\Delta}_{[t,j]} \cdot \hat{P}_t$$

ESTIMATED ASSIGNMENT MAP

$$f_{[j]} = \sum_{t=1}^{j} \hat{M}_{[t,j]} \cdot d_{[t]} + \varepsilon_{[j]}$$
DYNAMIC O-D ESTIMATION

Relationship between within-day dynamic traffic counts and O-D demand flows

- **Observed link flows (counts)** \( \hat{f}_{[j]} = \{\hat{f}_{[j]}\} \)
  
  flow counted on link \( l \) in time interval \( j \)

\[
\begin{align*}
  f_{[ij]} &= \sum_{t=1}^{j} \hat{M}_{[t,j]} \cdot d_{[t]} + \varepsilon_{[ij]}^{SIM} \\
  f_{[ij]} &= \hat{f}_{[ij]} + \varepsilon_{[ij]}^{OBS}
\end{align*}
\]

\[
\begin{align*}
  \hat{f}_{[ij]} &= \sum_{t=1}^{j} \hat{M}_{[t,j]} \cdot d_{[t]} + \varepsilon_{[ij]}
\end{align*}
\]

where \( \varepsilon_{[ij]} = \varepsilon_{[ij]}^{SIM} - \varepsilon_{[ij]}^{OBS} \)

\( \varepsilon = \) vector of random error terms due to:

- assignment model errors
- counting errors
DYNAMIC O-D ESTIMATION
Dynamic Estimators of O-D trip matrices

• SIMULTANEOUS ESTIMATORS
Computing the whole set of time-dependent O-D matrices by using counts over all intervals simultaneously

\[
\hat{d}_{[1]}^*, \ldots \hat{d}_{[j]}^*, \ldots \hat{d}_{[n]}^*
\]

\[
\hat{f}_{[1]}^*, \ldots \hat{f}_{[j]}^*, \ldots \hat{f}_{[n]}^*
\]
DYNAMIC O-D ESTIMATION
Dynamic Estimators of O-D trip matrices

• SIMULTANEOUS ESTIMATORS

Estimation of the whole set of demand vectors \( (\hat{d}_1^*, \ldots, \hat{d}_{nj}^*) \):

\[
(\hat{d}_1^*, \ldots, \hat{d}_{nj}^*) = \arg\min_{x_1 \geq 0, \ldots, x_{nj} \geq 0} z_1(x_1, \ldots, x_{nj}; \hat{d}_1, \ldots, \hat{d}_{nj}) + \\
+ z_2(x_1, \ldots, x_{nj}; \hat{f}_1, \ldots, \hat{f}_{nj})
\]

\( \hat{d}_{ij}^* \): initial information about O/D vector \( d_i \)

\( z_1(.) \) \( z_2(.) \): functions to be defined, depending on the chosen estimator

**GLS estimator**

\[
z_1(.) = \sum_{j=1}^{n} (x_j - \hat{d}_j)^T Z_j^{-1} (x_j - \hat{d}_j)
\]

\[
z_2(.) = \sum_{j=1}^{n} \left( \sum_{t=1}^{j} \hat{M}_{[t,j]} x_t - \hat{f}_j \right)^T W^{-1} \left( \hat{M}_{[t,j]} x_t - \hat{f}_j \right)
\]
OUTLINE

• PRELIMINARY CONSIDERATIONS

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QUASI DYNAMIC O-D ESTIMATION in within-day dynamic contexts

Given \( n_c \) centroids, \( n_l \) counted links and a period \( \tau \) including \( n_j \) time slices, the generic o-d flow for the time slice \( j \) may be expressed as the product between the demand generated by \( o \) during the time slice \( j \), \( g^j_o \), and the fraction going to destination \( d \) moving from \( o \) within the time slice \( j \), \( p^j_{d|o} \)

The factors affecting \( g^j_o \) are inherently within-day time varying, while the factors affecting \( p^j_{d|o} \) are more stable across different time slices.

\[
\frac{\partial g^j_o}{\partial j} \gg \frac{\partial p^j_{od}}{\partial j}
\]

The distribution probability \( p^j_{d|o} \) of the \( n_j \) time slices \( j \) within \( T \) may be reasonably approximated by its average \( p^{\tau(j)}_{d|o} \) over \( T \)

\[
d^j_{od} = g^j_o p^j_{d|o} \approx g^j_o p^{\tau(j)}_{d|o} = d_{od}^{j,qd}
\]
QUASI DYNAMIC O-D ESTIMATION in within-day dynamic contexts

Intrinsic Error

The intrinsic bound is the difference between the demand flow $d^i_{od}$ and the corresponding quasi-dynamic flow $d^{j,qd}_{od}$

\[ ie^j_{od} = d^j_{od} - d^{j,qd}_{od} \]
QUASI DYNAMIC O-D ESTIMATION in within-day dynamic contexts

The main problems in O-D estimation with traffic counts depends on the balance between the nr of equation and the nr of unknown

The quasi-dynamic assumption allows reducing the number of unknowns:
- generation profiles for each origin and time slice: $n_c \times n_j$ unknowns;
- average (within T) distribution shares: $nc^2$ unknowns
while the equations are $n_l \times n_j$

Playing on the lenght of T equations and unknown can be balanced
### QUASI DYNAMIC O-D ESTIMATION in within-day dynamic contexts

![Graphical representation of counted flows](image)

<table>
<thead>
<tr>
<th>Time Slices</th>
<th>Sensors</th>
<th>Equations</th>
<th>Unknowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>6</td>
<td>2</td>
</tr>
</tbody>
</table>

#### Example Equations

<table>
<thead>
<tr>
<th>$d_1$</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

- $d_{1[1]} = 9$
- $d_{2[1]} = 8$

<table>
<thead>
<tr>
<th>$d_2$</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

- $d_{1[2]} = 12$
- $d_{2[2]} = 12$
- $p(3/1) = 2/3$
- $p(3/2) = 1/4$

<table>
<thead>
<tr>
<th>$d_3$</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>12</td>
</tr>
</tbody>
</table>

- $d_{1[3]} = 15$
- $d_{2[3]} = 16$
QUASI DYNAMIC O-D ESTIMATION
in within-day dynamic contexts

GLS-based quasi-dynamic o-d flows estimator

\[
d^* = \arg \min_{x \in S} \left[ z_1(\mathbf{x}, \hat{\mathbf{d}}) + z_2(\mathbf{f}(\mathbf{x}), \hat{\mathbf{f}}) \right]
\]

\[
\{ \mathbf{g}^1, \ldots, \mathbf{g}^j, \ldots, \mathbf{g}^{n_j}; \mathbf{p}^1, \ldots, \mathbf{p}^{\tau}, \ldots, \mathbf{p}^{n_\tau} \} =
\]

\[
= \arg \min_{x^1 \ldots x^{n_j} \in S_x, \pi^1 \ldots \pi^{n_\tau} \in S_\pi} \left[ z_1(\mathbf{x}^1 \ldots \mathbf{x}^{n_j}, \pi^1 \ldots \pi^{n_\tau}, \hat{\mathbf{d}}^1 \ldots \hat{\mathbf{d}}^{n_j}) + z_2(\mathbf{f}(\mathbf{x}^1 \ldots \mathbf{x}^{n_j}, \pi^1 \ldots \pi^{n_\tau}), \hat{\mathbf{f}}^1 \ldots \hat{\mathbf{f}}^{n_j}) \right]
\]

\[
\{ \mathbf{g}^1, \ldots, \mathbf{g}^j, \ldots, \mathbf{g}^{n_j}; \mathbf{p}^1, \ldots, \mathbf{p}^{\tau}, \ldots, \mathbf{p}^{n_\tau} \} =
\]

\[
= \arg \min_{\substack{x^j \geq 0, \forall o, \forall j \in T; 0 \leq \pi_{d, p}^j \leq 1, \forall p \in d^j, \forall \tau \in T; \sum_{d} \pi_{d, p}^j = 1, \forall o, \forall \tau \in T}} \left\{ \sum_{j=1}^{n_j} \sum_{od=1}^{n_{od}} \left( \frac{x^j \cdot \pi_d^\tau(j) - \hat{d}_od^j}{\sigma_{od}^j} \right)^2 + \sum_{j=1}^{n_j} \sum_{l=1}^{n_l} \left( \frac{\sum_{j'=j, od=1}^{n_{od}} m^j_{j'} x^j_{o} \cdot \pi_d^\tau(j') - \hat{f}_l^j}{\sigma_l^j} \right)^2 \right\}
\]
QUASI DYNAMIC O-D ESTIMATION

Real test site of A4 - A23 motorways in North-Eastern Italy

- Full test site with 17 origins, 124 links and 272 o-d pairs
- The A4 branch between Palmanova and Trieste was eliminated and replaced by a virtual junction close to the A4-A23 intersection
- Two independent closed systems (one per carriageway) made by 13 origins, 91 o-d pairs and 49 links each

- The entrance/exit toll system allows to obtain the true 10 minutes o-d flows (cars)
QUASI DYNAMIC O-D ESTIMATION in within-day dynamic contexts

Scheme of the experiment

Ex: spreading: equiprobabilistic distribution model
QUASI DYNAMIC O-D ESTIMATION

Description of the experiments on the real test site of A4 - A23 motorways in North-Eastern Italy

✓ Perturbation of the observed o-d flows to obtain the seed o-d flows
  • the observed generation profiles are perturbed using a coefficient of variation of 0.3
  • uniform distribution shares across all destinations are assumed

✓ Estimation/updating on the basis of a subset of 15 link counts
  • the subset is chosen by means of the max flow method proposed by Yang and Zhou (1998) amongst the observed link flows
  • the observed link flows are calculated through the dynamic network loading of the observed o-d flows
  • error-free link counts, error-free assignment map

✓ The quality of the tested estimator is measured by comparing the updated o-d flows with the observed o-d flows
  • standard goodness-of-fit measures are used
QUASI DYNAMIC O-D ESTIMATION

Experiments on the real test site of A4 - A23 motorways in North-Eastern Italy

Tested estimators

✓ QD-GLS estimator
  • the distribution shares are kept constant for the whole day

✓ Static GLS estimator
  • 24 daily static estimate/updates (a duration $T_s=1$ hour is assumed)

✓ Simultaneous dynamic estimator

✓ Kalman filter estimator
  • three different experiments are carried out depending on the type of seed o-d flows
QUASI DYNAMIC O-D ESTIMATION

Experiments on the real test site of A4 - A23 motorways in North-Eastern Italy

Results

<table>
<thead>
<tr>
<th>day</th>
<th>Seed matrix</th>
<th>Simultaneous updating</th>
<th>Quasi-dynamic (τ=T=24h)</th>
<th>Kalman filter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>absolute</td>
<td>% reduction</td>
<td>absolute</td>
</tr>
<tr>
<td>MO April 14th</td>
<td>32.9</td>
<td>20.9</td>
<td>-37%</td>
<td>2.41</td>
</tr>
<tr>
<td>TU April 15th</td>
<td>35.1</td>
<td>21.0</td>
<td>-40%</td>
<td>2.38</td>
</tr>
<tr>
<td>WE April 16th</td>
<td>36.7</td>
<td>21.6</td>
<td>-41%</td>
<td>2.59</td>
</tr>
<tr>
<td>TH April 17th</td>
<td>38.7</td>
<td>21.9</td>
<td>-44%</td>
<td>2.74</td>
</tr>
<tr>
<td>FR April 18th</td>
<td>48.2</td>
<td>23.3</td>
<td>-52%</td>
<td>3.91</td>
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<td>1.55</td>
<td>-30%</td>
<td>0.63</td>
</tr>
</tbody>
</table>

Effectiveness of the QD-GLS estimator: significant reduction with respect to the initial perturbed seed o-d flows

The performances of the Kalman filter are entirely dependent on the quality of the seed o-d flows

The simultaneous estimator provides a noteworthy reduction but is outperformed by the QD

Experiments on the real test site of A4 - A23 motorways in North-Eastern Italy

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<td>0.59</td>
</tr>
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<td>FR April 18th</td>
<td>2.22</td>
<td>1.55</td>
<td>-30%</td>
<td>0.63</td>
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</table>

Effectiveness of the QD-GLS estimator: significant reduction with respect to the initial perturbed seed o-d flows

The performances of the Kalman filter are entirely dependent on the quality of the seed o-d flows

Experiments on the real test site of A4 - A23 motorways in North-Eastern Italy

Results

<table>
<thead>
<tr>
<th>day</th>
<th>Seed matrix</th>
<th>Simultaneous updating</th>
<th>Quasi-dynamic (τ=T=24h)</th>
<th>Kalman filter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>absolute</td>
<td>% reduction</td>
<td>absolute</td>
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<tr>
<td>MO April 14th</td>
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<td>48.2</td>
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</table>

Effectiveness of the QD-GLS estimator: significant reduction with respect to the initial perturbed seed o-d flows

The performances of the Kalman filter are entirely dependent on the quality of the seed o-d flows

Experiments on the real test site of A4 - A23 motorways in North-Eastern Italy

Results

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<tbody>
<tr>
<td></td>
<td></td>
<td>absolute</td>
<td>% reduction</td>
<td>absolute</td>
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<td>2.23</td>
<td>1.67</td>
<td>-25%</td>
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</tr>
<tr>
<td>FR April 18th</td>
<td>2.22</td>
<td>1.55</td>
<td>-30%</td>
<td>0.63</td>
</tr>
</tbody>
</table>

Effectiveness of the QD-GLS estimator: significant reduction with respect to the initial perturbed seed o-d flows

The performances of the Kalman filter are entirely dependent on the quality of the seed o-d flows
### QUASI DYNAMIC O-D ESTIMATION

Experiments on the real test site of A4 - A23 motorways in North-Eastern Italy

Results of hourly o-d estimates obtained with the static GLS estimator and by aggregating simultaneous, QD-GLS and Kalman filter estimates

<table>
<thead>
<tr>
<th>Day</th>
<th>Seed Matrix</th>
<th>StaticUpdating</th>
<th>Simultaneous Updating</th>
<th>Quasi-Dynamic (T=24h)</th>
<th>Kalman Filter</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>absolute % reduction</td>
<td>absolute % reduction</td>
<td>intrinsic error</td>
<td>updating</td>
</tr>
<tr>
<td>MO</td>
<td>April 14th</td>
<td>1047</td>
<td>734 -30%</td>
<td>681 -35%</td>
<td>27 155 -85%</td>
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<tr>
<td>TU</td>
<td>April 15th</td>
<td>1101</td>
<td>795 -28%</td>
<td>677 -33%</td>
<td>28 170 -85%</td>
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<tr>
<td>WE</td>
<td>April 16th</td>
<td>1151</td>
<td>824 -31%</td>
<td>702 -43%</td>
<td>33 192 -84%</td>
</tr>
<tr>
<td>TH</td>
<td>April 17th</td>
<td>1221</td>
<td>839 -28%</td>
<td>693 -40%</td>
<td>32 187 -84%</td>
</tr>
<tr>
<td>FR</td>
<td>April 18th</td>
<td>1539</td>
<td>959 -38%</td>
<td>679 -56%</td>
<td>58 394 -74%</td>
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</table>

<table>
<thead>
<tr>
<th>Day</th>
<th>Seed Matrix</th>
<th>StaticUpdating</th>
<th>Simultaneous Updating</th>
<th>Quasi-Dynamic (T=24h)</th>
<th>Kalman Filter</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>absolute % reduction</td>
<td>absolute % reduction</td>
<td>intrinsic error</td>
<td>updating</td>
</tr>
<tr>
<td>MO</td>
<td>April 14th</td>
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<td>0.33 0.79 -62%</td>
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<td>1.80 -15%</td>
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<td>WE</td>
<td>April 16th</td>
<td>2.09</td>
<td>1.77 -15%</td>
<td>1.62 -23%</td>
<td>0.35 0.84 -60%</td>
</tr>
<tr>
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<td>April 17th</td>
<td>2.08</td>
<td>1.73 -17%</td>
<td>1.58 -24%</td>
<td>0.34 0.83 -60%</td>
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<tr>
<td>FR</td>
<td>April 18th</td>
<td>2.09</td>
<td>1.65 -21%</td>
<td>1.39 -34%</td>
<td>0.41 1.06 -49%</td>
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</tbody>
</table>

The QD-GLS estimator outperforms both the static and the simultaneous estimators.
QUASI DYNAMIC O-D ESTIMATION

Experiments on the real test site of A4 - A23 motorways in North-Eastern Italy

Distances between the observed link flows and the link flows obtained assigning the updated o-d flows

The intrinsic bound is the distance between the observed link flows and the flows obtained assigning the “true” quasi-dynamic o-d flows

The QD-GLS estimator is very robust on the hold-out sample, outperforming the simultaneous estimator and allowing the Kalman filter to obtain very effective results

The simultaneous estimator outperforms others for counted links

<table>
<thead>
<tr>
<th>day</th>
<th>all links MSE</th>
<th>counted links MSE</th>
<th>hold-out sample MSE</th>
<th>counted links RMSE</th>
<th>hold-out sample RMSE</th>
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<td></td>
<td>QD-GLS</td>
<td>Kalman</td>
<td>QD-GLS</td>
<td>Kalman</td>
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<tr>
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<td>369 24 80</td>
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<td>9 49 122</td>
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<td>70 46 264 31 398 129</td>
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<td>20 32 136</td>
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<td>35 359 126</td>
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<table>
<thead>
<tr>
<th>day</th>
<th>all links RMSE</th>
<th>counted links RMSE</th>
<th>hold-out sample RMSE</th>
<th>counted links</th>
<th>hold-out sample</th>
</tr>
</thead>
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<td></td>
<td>QD-GLS</td>
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<td>QD-GLS</td>
<td>Kalman</td>
<td>QD-GLS</td>
</tr>
<tr>
<td>MO April 14th</td>
<td>0.50 0.13 0.23</td>
<td>- - -</td>
<td>0.04 0.08 0.12</td>
<td>- - -</td>
<td>1.31 0.21 0.61</td>
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<tr>
<td>TU April 15th</td>
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<td>0.04 0.08 0.12</td>
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<td>1.43 0.23 0.70</td>
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<tr>
<td>WE April 16th</td>
<td>0.53 0.13 0.30 0.10</td>
<td>0.48 0.26</td>
<td>0.04 0.08 0.13</td>
<td>0.02 0.04 0.04</td>
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<tr>
<td>TH April 17th</td>
<td>0.49 0.13 0.29 0.10</td>
<td>0.49 0.27</td>
<td>0.03 0.08 0.12</td>
<td>0.02 0.04 0.11</td>
<td>1.29 0.22 0.64</td>
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<tr>
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<td>0.17 0.14 0.34 0.11</td>
<td>0.41 0.24</td>
<td>0.08 0.09 0.20</td>
<td>0.04 0.05 0.11</td>
<td>1.14 0.25 0.61</td>
</tr>
</tbody>
</table>
CONCLUSIONS

STATIC
✓ easier to implement (static assignment matrix)
✓ faster to compute

DYNAMIC
✓ better estimators of static flows!
✓ needed for dynamic assignment
✓ quasi-dynamic outperforms simultaneous
✓ quasi-dynamic computationally more time-demanding
CONCLUSIONS

Innovative info sources to estimate O-D flows

Traditional surveys

Future mobility services

Floating Car Data

Inition estimate of O/D flows

Traffic Counts

O/D correction with traffic counts

Corrected O/D flows